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ABSTRACT

Compared were the effects of two experimental arithmetic treatments, called Laboratory and Textbook, upon achievement and attitude development of fourth grade children. Prior to the study, the experimenter employed task analysis procedures to develop hierarchies of skills for the four operations on whole numbers. During the instructional phase of 29 days, the 73 students in the Laboratory group performed teacher-designed instructional activities that involved extensive use of games and manipulation of physical objects by children. The 74 students in the Textbook group were assigned traditional work from an arithmetic textbook. Children with behavior problems and learning difficulties in arithmetic were identified. At the conclusion of the experimental phase, three measures of children's arithmetic achievement were obtained: computation skills, knowledge of concepts, and application of processes. Measures were also taken of children's attitudes toward arithmetic, school, reading, spelling, and science. As measured by a commercial achievement test, no significant differences between treatments were found for each of the three areas of arithmetic achievement investigated. On experimenter-designed achievement tests, significant differences between treatments were found for Laboratory treatment of concepts and Textbook treatment of computation skills at the .05 and .01 levels, respectively. (Author/CB)

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DIFFERENTIAL EFFECTS OF ACTIVITY-ORIENTED VS. TEXTBOOK-ORIENTED MATHEMATICS INSTRUCTION FOR ELEMENTARY SCHOOL CHILDREN WITH LEARNING AND BEHAVIOR PROBLEMS

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by

William Dunlap
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1971

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COVER PAGE

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CHAPTER ONE

NATURE OF THE PROBLEM

One often hears comments from teachers and parents, such as, "John is having trouble with arithmetic" or "Bill can't do division problems." Such comments, while indicative of an affective set, do not really describe a child's problem. Can't Bill do any division problems or are there certain kinds of problems he misses? More information is needed before one can identify misunderstood division concepts which are causing the child to be unsuccessful.

In other instances, a child may successfully implement an algorithm on one day but not on the next. Reasons for this failure to consistently apply algorithms are often not readily apparent to the teacher. For example, did the child assimilate and accommodate the algorithm through rote processes (Phillips, 1969)? Further, one might ask if he attained the prerequisite skills which are necessary for him to meaningfully and successfully implement the algorithm? To adequately analyze the situation, teachers must answer these and other questions, for such analysis is necessary to help them choose appropriate instructional activities.

Diagnostic and Remedial Programs

Teachers have implemented many different programs to diagnose, analyze, and eliminate a child's learning difficulties in arithmetic. For instance, Harvey (1953) designed a diagnostic and remedial arithmetic program which was essentially cyclic in nature: testing - reteaching - retesting. Teachers administered a diagnostic test and prepared worksheets containing paper-pencil exercises for children on the basis of their performance on the test. After children completed a specific number of worksheets, diagnostic tests were again administered.

A different program was described by Cotter (1967). Diagnostic and remedial procedures were implemented by specialized teachers in a small group environment. Children studied the regular arithmetic curriculum. However, concrete materials and illustrations were used extensively by both children and teachers.

In another program, Otto (1927) placed children with similar arithmetic deficiencies in the same group. He divided the arithmetic period into three parts. During one part, children drilled on processes in which they were weak. A practice sheet on each operation was completed during another part of the period. Later, children drilled on flash cards and completed worksheets on basic combinations.

Another diagnostic and remedial arithmetic program has been used by Higgins and Rusch (1965). They used a programmed arithmetic textbook and associated diagnostic tests.

Many programs require the separation of children with learning difficulties from their peers for special instruction. However, Jacqueline (1941) had children participate in the regular arithmetic class. Then remedial instruction was given at a different time of day on a regular basis, such as after school or during a free period.

In contrast to these programs which often made extensive use of paper-pencil and drill activities, Lerch and Kelly (1966) used instructional activities that involved inquiry, exploration, discovery, and manipulative aids. Further, instruction was grouped into topical units, such as area and volume, number theory, or exponents.

A similar program was devised by Fogler (1953). The remedial program was structured by a workbook; however, concrete, manipulative aids were extensively used to help children correctly develop concepts.

For most of these diagnostic and remedial arithmetic programs, diagnostic procedures consisted of the administration of a paper-pencil test. However, Weaver (1955) and Burge (1934) have successfully employed interview techniques for diagnosis of children's learning difficulties. In interviews teachers may systematically ask questions or children may "think-out-loud" to reveal conditions that would otherwise be undetected.

Further, Smith (1969) has suggested that children manipulate objects to demonstrate their thought processes and knowledge of mathematical principles. Children may manipulate objects in an interview or when completing a written test.

Model For a Diagnostic and Remedial Program

Teachers employ many different procedures to diagnose and eliminate children's arithmetic learning difficulties. Regardless of the diagnostic and remedial procedure, the teacher needs a model or map of skills which are necessary for a child to successfully grasp a mathematical principle. To develop a model or map of skills, Glennon and Callahan (1968) have suggested that teachers apply task analysis to a particular segment of arithmetic knowledge.

In task analysis, the teacher begins her analysis with the final task or desired terminal behavior. Then the question, "What particular skills must the child possess to be able to successfully perform this task?" is asked. This enables the teacher to identify and list particular (subordinate skills) which are necessary to the acquisition of the final task. Then the same question is applied to each of the subordinate skills. Levels of subordinate skills are established with each level becoming increasingly more basic.

Implicit within the use of task analysis is the assumption that certain skills must be assimilated and accommodated by the child prior to his successfully acquiring the knowledge and use of other "higher level" skills. To test this assumption, Gagne (1962) identified the prerequisite skills necessary for a child to successfully master a mathematical principle. In his study, he found that children, who did not grasp the prerequisite skills, were unsuccessful at demonstrating their knowledge of the mathematical principle being studied. Thus, this study lends credence to the possibility that children, who are having difficulties in arithmetic, may not have knowledge of prerequisite skills.

Defining a Skill

Once a hierarchy of subordinate skills has been contructed through task analysis, each skill in the hierarchy should be behaviorally defined to aid in designing instructional activities. This may be accomplished by formulating a statement describing the behavior which the child must exhibit in order for the teacher to ascertain the child's mastery of the skill. Mager (1962) has suggested that behavioral objectives contain three distinct and important parts:

1. Identify the terminal behavior by name. Specify the behavior that will be accepted as evidence that the learner has acquired the skill.
2. Describe the important conditions under which the behavior will be expected to occur.
3. Specify the criteria of acceptable performance by describing how well the learner must perform to be considered acceptable.

From these behavioral objectives, a remedial program for each skill may be planned. Also, teachers are able to determine when the child has acquired competency with a particular skill.

Levels of Learning and Testing

In designing a diagnostic and remedial program, Gagne (1965) has suggested that teachers must be aware of different levels of learning: (1) rote or associative learning, (2) concept learning, (3) principle level and (4) problem solving learning. Test questions and instructional activities may be devised to determine the level at which a child is presently functioning and to help the child proceed to higher levels of learning.

Models and Manipulative Materials

Some educators believe that remedial programs should contain instructional activities that involve the use of manipulative materials. For instance, Bruner (1964) has postulated that a child integrates a concept into his cognitive structure in three stages: (1) enactive, (2) iconic, and (3) symbolic. At the enactive stage, a child must use physical materials which he manipulates to learn about the concept being taught. He is unable to form a mental image of the concept. After the child has manipulated physical objects for a time, he becomes able to form a mental image of the concept. The child is now at the iconic stage. Manipulations are performed on the mental image or pictures of the concept. Later, at the symbolic stage, the child thinks of the concept in terms of some abstract symbols which may or may not have any physical relationship to the concept except that he has chosen the symbols to represent the concepts.

Through extensive study, Piaget (Phillips, 1969) distinguished four stages of cognitive development: (1) sensory-motor, (2) pre-operational, (3) concrete operations and (4) formal operations. The middle two stages, pre-operational and concrete operations, extend over the years that a child is in the typical elementary school. A child's thought processes are based upon observation and experience with objects in the physical world. It is important that children be provided an instructional environment in which they may manipulate physical objects for themselves and find their own answers. Learning comes not from the objects themselves, but rather the child's physical action on the objects. This implies that the child should perform the manipulation of objects and not just be shown the manipulation by the teacher.

From the consideration of the cognitive development theories given by Bruner and Piaget, it becomes quite evident that physical models or objects should receive extensive use in a remedial instructional program. When initiating a child's experience with a particular skill, activities should be planned which make use of physical models that enable the child to formulate a mental representation of the skill. When engaged in the activities, it is important that the child perform the physical action on the objects.

PURPOSE OF STUDY

In contrast to diagnostic and remedial arithmetic programs currently being employed by some educators, the experimenter believes that an effective diagnostic and remedial arithmetic program involves the following:

1. Develop a hierarchy of subordinate skills through task analysis of the arithmetic concepts suggested by the child's performance.
2. Define each subordinate skill in behavioral terms.
3. Develop a test to measure a child's competencies regarding each skill in the hierarchy. Test questions should reflect the four levels of learning: associative, conceptual, principle and problem solving.
4. Administer the test to children individually or in small groups, two or three children.
5. Formulate tentative hypotheses of the child's learning difficulties.
6. Design instructional activities reflecting levels of learning and manipulation of physical objects by children.
7. Evaluate the child's progress and make any adjustments in the instructional program that are necessary.

Therefore, the purpose of this study was to determine the effect that diagnostic and remedial arithmetic programs have upon the arithmetic achievement and attitude development of fourth grade children who have arithmetic learning difficulties.

To accomplish this purpose, two experimental treatments were used and compared:

1. Laboratory Treatment - Diagnosis was based upon a child's performance on a experimenter-designed diagnostic arithmetic test. Instructional activities made extensive use of models, manipulative aids, games and discovery of patterns.
2. Textbook Treatment - Diagnosis was based upon a child's performance on a standardized diagnostic arithmetic test. Instructional activities consisted of paper-pencil exercises found in a textbook.

Thus, two major questions to be answered by this study are:

1. "What is the effect of these two diagnostic and remedial treatments upon the computational skills, knowledge of arithmetic concepts, and problem solving ability of children as shown by : (a) Comprehensive Test of Basic Skills: Arithmetic Test, and (b) experimenter-designed arithmetic achievement test?"
2. "What is the effect of these two diagnostic and remedial treatments upon the attitudes of children toward arithmetic, school, reading, spelling and science as shown by an experimenter designed test using Osgood's Semantic Differential as a model?"

SETTING OF THE STUDY

This study was conducted from March 30, 1970, until June 5, 1970, in fourth grade classrooms in the Bethel School System, Eugene School System and Springfield School System. Table 1.01 gives the number of schools and students for each of the three school districts.

The Bethel School District is predominantly rural with some sub-divisions of developer-built homes. Most of the houses are valued in the $15,000-$20,000 range. The area has little commercial and industrial property. Thus, its main source of finance comes from property tax on residential dwellings and state aid.

Since the Bethel School District is largely rural, several residents receive income from agricultural enterprises. Further, many people are employed by the logging industry or by commercial enterprises in Eugene and Springfield. Many of these people are employed in skilled or semi-skilled trades which do not require a high level of formal education. Therefore, the income range for residents of Bethel School District could be considered as average to low average.

In contrast to Bethel, the Eugene School District contains a large proportion of developer-built houses and commercial property with only a small amount of rural property. The cost of most houses would be in the $20,000-$30,000 range. The Eugene School District receives a large amount of their funds from property tax on commercial and industrial property, but a substantial property tax on residential dwellings still exists.

The income level for the residents of the Springfield School

District could be considered as average to low average. Many residents receive income from skilled and semi-skilled trades which are required by commerical enterprises in Eugene and Springfield. Also, the logging industry employs many people from this district.

Lately, the three school districts have not enjoyed widespread support from the local taxpayers. Voters in each district have defeated at least two budget proposals each year before finally approving an operating school budget.

PLAN OF THE STUDY

The population of the study included two elementary schools in the Bethel School District, nine elementary schools in the Eugene School District and one elementary school in the Springfield School District. The administration in each district selected the schools because of: (1) cooperativeness of the staff, (2) available space and facilities for small group activities and (3) large number of students which were having learning difficulties in arithmetic, reading and language arts.

Twenty-four experienced teachers participated in the study. Each of these teachers were also participants in the Experienced Teacher Fellowship Program. These teachers received training in developing a hierarchy of arithmetic skills, writing behavioral objectives, constructing diagnostic tests and preparing instructional activities which made extensive use of physical models manipulated by children.

To initiate the study, the experimenter developed a hierarchy of arithmetic skills for the operations of addition, subtraction, multiplication, and division on whole numbers. Each skill in the hierarchy was behaviorally defined.

A diagnostic arithmetic test was then prepared by the experimenter to identify those skills in the hierarchy which a child has not assimilated and accommodated into his cognitive structure. This test contained exercises requiring children to: (1) manipulate physical objects illustrating a mathematical principle, (2) write a number sentence describing the action performed on objects by the examiner, and (3) solve problems using pencil-paper. Teachers administered the diagnostic arithmetic test to fifth grade children with learning difficulties in arithmetic. From these results, the test was revised by the experimenter with the assistance of the teachers for use in the study.

At this point in the study, two ExTFP Fellows were assigned to each of the schools involved in the study. Within each school, the

ExTFP Fellows and fourth grade teachers selected twelve fourth grade children for participation in the study. Children were selected on the basis of their performance on daily arithmetic assignments, teacher-constructed arithmetic tests, and standardized achievement tests. These children were randomly assigned to the Laboratory Treatment and Textbook Treatment by drawing their names from a box. Also, the ExTFP Fellows randomly assigned themselves to treatments.

Prior to the instructional phase of the study, the Comprehensive Test of Basic Skills: Arithmetic Battery, Stanford Diagnostic Arithmetic Test and experimenter-designed diagnostic arithmetic test were administered to each child in the study.

Two experimental treatments assigned during the study were:

1. Laboratory Treatment - The experimenter-designed diagnostic arithmetic test indicated those skills in the arithmetic hierarchy which the child had not mastered. Each child began instructional activities related to the lowest level arithmetic skills which he could not successfully perform. The instructional activities, designed by ExTFP Fellows, made extensive use of the levels of learning, behavioral objectives, manipulation of physical objects by children and games.

2. Textbook Treatment - A child was given an arithmetic textbook whose grade level corresponded to his grade equivalent score on the Stanford Diagnostic Arithmetic Test. During the instructional phase, these children performed only the activities in their arithmetic textbook and those suggested by the accompanying teacher's guide. These were predominantly paper and pencil instructional activities.

At the conclusion of the instructional phase, the Comprehensive Test of Basic Skills: Arithmetic, experimenter-designed arithmetic achievement test and attitudes scales were administered to every child participating in the study.

Procedures for sample selection, instructional procedures and content of tests are discussed in further detail in Chapter III.

TABLE 1.01

NUMBER OF SCHOOLS AND STUDENTS IN THE DISTRICTS PARTICIPATING IN THE STUDY

District	Elem. Schs.	Jr. Highs	Sr. Highs	Elem. Students	Jr. High Students	Sr. High Students
Bethel	5	2	1	1,822	852	736
Eugene	32	8	4	11,073	5,308	4,721
Springfield	14	4	2	5,100	2,486	2,180

HYPOTHESES

The purpose of this experiment was to determine the effects of two diagnostic and remedial arithmetic programs upon the arithmetic achievement and attitude development of fourth grade children with learning difficulties in arithmetic. To do this, two experimental treatments were used and their effects on children's achievement determined through tests of: (1) computational skills, (2) arithmetic concepts, and (3) applications. Also, the same two experimental treatments were used to identify their effects upon children's attitudes as measured by attitudes scales for: (1) arithmetic, (2) school, (3) reading, (4) spelling, and (5) science.

Thirteen null hypotheses were formulted for testing. They are as follows:

1. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

2. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the mathematical concepts test of the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

3. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the applications test of the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

4. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the total battery of the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

5. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the manipulation test of the experimenter designed achievement test.

6. No significant difference exists among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the experimenter designed achievement test.

7. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the applications test of the experimenter designed achievement test.

8. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the total battery of the experimenter designed achievement test.

9. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude towards arithmetic.

10. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude towards school.

11. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude towards reading.

12. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude towards spelling.

13. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude towards science.

CHAPTER II

REVIEW OF LITERATURE

There are essentially four areas of interest in this study: (1) diagnosis, (2) remedial instruction, (3) use of models and manipulative aids, and (4) attitude of students. This chapter is a review of journal articles relevant to each of the four areas. Several articles represent opinions of educational leaders concerning one or more facets of this study while other articles report research which has been completed.

Carefully designed and interpreted research studies of the effects of diagnostic and remedial instructional programs are rare and many are quite dated. However, research studies concerning the effects of manipulative aids and children's attitudes are more recent and better designed.

DIAGNOSIS OF ARITHMETIC DEFICIENCIES

Opinions Regarding Diagnostic Teaching

Originally educators only thought of diagnosis as it applied to children with learning difficulties. In contrast to this belief, Hunter (1966) feels that diagnosis should not be reserved for the educationally "sick". Rather, diagnosis should be an intrinsic part of the teaching act for all learners since diagnostic procedures contribute data for answering these questions:

1. What objective is appropriate for this learner?
2. What is his present status in relation to that objective?
3. What is the next learning step in the attainment of that objective?
4. What can the teacher do to help the child make that step efficiently and economically?

Similarly, Suchman (1967) maintains that diagnostic teaching gets directly to what the learner knows and accepts, how he knows, and why he accepts it. He stresses that the teacher, through sequential questions, keeps pushing the learner to examine his own knowledge so that he is aware of his own cognitive structure.

While elaborating upon this viewpoint, Ponder (1968) suggested that diagnostic teaching focuses on ordering learning goals so that any learner can set out to reach the goal which suits his current needs. The establishment of learning goals may be accomplished by setting up necessary sequences and then breaking them down into manageable steps. Learning goals are translated into behavioral terms so that sequential learning cotinua for accomplishing them can be established. Furthermore, Utsey (1965) feels that the teacher's task is one of accepting children and continuously assessing their levels of achievement in order to provide more effective guidance in the learning process. He believes that informal diagnosis yields information about a child's instructional level as well as a great deal of significant data regarding children's attitudes, working habits, and etc.

As another proponent of diagnostic teaching, Eisele (1967) stated that the ultimate improvement in teaching and learning will only be realized when teachers assume the role of learning diagnosticians, with a basic committment to prescribing learning activities which will enable the pupil to meet his individual needs as a learner. His concept of diagnostic teaching consists of four phases: (1) determining the needs of individual learners, (2) specifying instructional objectives, (3) prescribing and guiding learning activities to help the pupil achieve his objectives, and (4) assisting in the process of evaluating pupils.

Further support for diagnostic teaching was given by Goodlad (1966). He feels that creative diagnosis and prescription by teachers must be facilitated if educators are ever to talk less and act more with respect to meeting the needs of individual learners. Moreover, in order for diagnosis and prescription to be effective, the following conditions must be realized:

1. Develop systems for collecting data necessary for diagnosis.
2. Teachers must become adept at interpreting data.
3. Specialists must develop curriculum guides based on taxonomical analysis of behaviors and concepts.
4. Curriculum makers must stock teachers' pedagogical pharmarcies with the widest range of instructional alternatives.

As a result of diagnostic teaching, Wayson (1967) believes that diagnostic procedures should result in vastly improved productivity on the part of pupils. However, this increased productivity requires additional time and personnel who have the ability and incentive to apply diagnostic procedures.

Diagnostic Techniques in Arithmetic

Educators may employ certain techniques to diagnose a child's learning difficulties. These diagnostic techniques may involve: (1) standardized diagnostic tests, (2) teacher-constructed diagnostic tests, (3) standardized achievement tests or (4) interviews.

Standardized diagnostic tests are extensively used in diagnostic programs. However, many of the available standardized diagnostic arithmetic tests were copyrighted over ten years ago. For example, four standardized diagnostic arithmetic tests which have been used in many research studies have the following copyright dates:

1.	Brueckner Diagnostic Arithmetic Tests	1926-43
2.	Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic	1925
3.	Brueckner Diagnostic Tests and Self Helps in Arithmetic	1955
4.	Los Angeles Diagnostic Tests: Fundamentals in Arithmetic	1925-47

Furthermore, Gronlund (1969) believes that several reservations should be kept in mind when selecting and using diagnostic tests: (1) the diagnostic procedure must be evaluated in light of the type of information desired, (2) diagnostic tests indicate the typical errors a pupil makes but they do not indicate the causes of the errors, and (3) diagnostic tests tend to have low reliability because of the relatively few items measuring each type of error.

On occasion, standardized achievement tests are used to gain data as a basis for diagnosis. Hunter (1967) says that standardized achievement tests give unusually vague information for diagnosing a child's learning difficulties. Teachers must use diagnostic instruments which show the precise point in a subject matter sequence that the student's knowledge leaves off.

In addition, some educators feel that standardized diagnostic tests do not yield applicable information. For instance, Ashlock (1967) believes that teachers need to identify the understandings and skills that are prerequisites to intelligent performance of the process he is trying to teach. However, he stresses that standardized diagnostic tests are not always the best way to diagnose learning difficulties. They may or may not include items for each understanding and skill prerequisite to the computational procedures being used in the mathematics program. Thus, teachers may find it necessary to construct diagnostic tests.

In supporting statements, Burns (1965) has indicated that teachers should purchase or construct tests for diagnostic procedures which contain problems corresponding to a particular skill or concept. Problems must be chosen that enable a teacher to pin point the particular skills that a child does not possess.

Similarly, Ross (1963) suggested that teachers construct an informal arithmetic inventory (IAI). When constructing an IAI, the first step is to establish a hierarchy of difficulty in computational skills. Later, verbal problems should be written which maintain the hierarchy of difficulty as established for computational skills.

Moreover, as an alternative to diagnostic tests that only emphasize computation, Smith (1969) has suggested that children manipulate objects to demonstrate their thought processes and knowledge of mathematical principles. For example, to determine a child's knowledge of correspondence, he would match an object in one set to an object in a second set. First, similar objects would be used, then objects which were dissimilar. As they are able to execute the task, place different number of objects in each set and ask which set contains more or less than the other set.

When constructing diagnostic tests, teachers must be concerned with reliability of the test. Grossnickle (1935) conducted a study to determine the reliability of diagnosis of errors. Two hundred students in each grade from five through twelve were given a written test on division. Then diagnosis was made from the student's performance on the test. Grossnickle found that a pupil must be given the opportunity to make at least three responses to each fact before the diagnosis was reliable.

Further credence was given this finding when Brueckner (1932) completed a study involving 327 pupils in the fifth grade. From his study Brueckner concluded that at least three examples of each type should be included in a test for individual diagnosis to be reliable, since errors on a single example are likely to be due to chance or mere accident.

The reliability of diagnosis problem was approached from another viewpoint when Olander (1933) conducted a study to determine the extent that it is possible for a teacher to ascertain by an examination of a pupil's test paper the needs of the pupil which should guide the administration of remedial instruction. Seven fourth grade students were individually administered a test. Each student worked out the answers on paper and verbally described their processes. Later, forty teachers analyzed the test papers and designated what they surmised were the thought processes of each pupil. The experimenter found that teachers were able to diagnose the causes of a pupil's errors in division with a high degree of accuracy. However, in addition, subtraction and multiplication, their success was rather limited: thirty three percent or less.

As an alternative technique for diagnosing learning difficulties, Weaver (1955) suggested that teachers use the interview technique. Two characteristics of an interview make it valuable as a diagnostic technique. First, for an interview conducted on an individual basis, children "think out loud" as they respond to specific quantitative situations and related questions. Secondly, the interview is structured around arithmetic problems which may cause children difficulty.

Likewise, Burge (1934) supports the use of interview techniques. He feels that systematic questioning will often reveal conditions that would otherwise be undetected. However, the success of the interview as a means of diagnosis depends primarily upon the development of a careful method of approach to the child.

Additional support was given the use of interview procedures when Brownell (1936) conducted a study comparing two diagnostic techniques; interview versus analysis of written responses. He used 245 fifth grade students in the study. The findings of the study were the following:

1. Both techniques were equally satisfactory only for the grosser types of diagnosis.

2. Analysis of written responses is preferred because of its convenience.

3. Interview technique is both more reliable and more valid when diagnosis is that of the processes and difficulties of individual children.

Diagnosis of Typical Arithmetic Errors

The most common method of making a diagnostic study was to give a test, either standardized or teacher constructed, covering the four fundamental processes and then to analyze, in detail, the errors made on the test. The object of this procedure was to discover the most common types of errors. Then remedial teaching was applied to eliminate errors which were most often committed.

A classic study of this nature was done by Buswell and John (1926). A total of 352 students in grades three through six were involved in the study. A test on whole numbers was administered to the children. Then the experimenters categorized the children's errors. There were thirty-three categories for errors in addition, twenty-three categories for subtraction, and forty-one categories for both multiplication and division. The most prevalent error for each operation was knowledge of combinations. The next most common error for each operation was: (1) carrying for addition, (2) borrowing for subtraction, (3) carrying for multiplication, and (4) subtraction for division.

When studying typical arithmetic errors involving whole numbers, Smith and Eaton (1939) used a mechanical master for displaying to groups of fourth graders the basic combinations in each operation. They found that addition facts were most thoroughly mastered at the fourth grade level, with zero combinations most frequently missed.

In another study concerned with errors made by children solving problems involving whole numbers, Roberts (1968) analyzed the written responses of 766 third grade children. From this analysis, he concluded that children's errors were most commonly caused by: (1) using the wrong operation, (2) computational errors, and (3) defective algorithms.

Similar results were found by Grossnickle and Snyder in a study involving 100 pupils in each of grades four through eight. Pupils took a test in each operation five different times after an interval of one to two days between successive tests. Besides finding that the zero combinations were most frequently missed, they found that most of the errors in addition and subtraction combinations were sporadic. They concluded that diagnosis made from the results of one test in these processes would be very unreliable. However, errors in multiplication and division facts were more constant.

When studying the errors committed by fifth and sixth grade children as they worked with fractions, Brueckner (1928) found that lack of comprehension of the process to use, inability to express fractions in simplest form, and difficulty in renaming improper fractions were the major causes of errors. Later, Morton (1924) and Schane (1938) substantiated these results.

Besides the analysis of errors in fractions, Brueckner (1928) analyzed the errors made by students in solving decimal problems. Results of this study showed that number combinations, placement of decimal point, borrowing and lack of comprehension of numerical values of decimals were major causes of error.

Moreover, Burge (1932) administered a test to 2577 children in grades four through six. He used the interview technique with low scoring children whose mental processes were not evident from their written work on the test. Buswell and John's list of most frequenterrors was used to categorize the errors made by the students. Burge found: (1) relatively few type errors and questionable habits of work could be analyzed with certainty from written responses, and (2) children tend to evolve questionable processes when skills are employed before the basic concepts are formed.

Additional information regarding arithmetic errors committed by children was given by William and Whitaker (1937), who conducted a study to determine the characteristic arithmetic deficiencies of children in grades four through eight. The study involved ten children from each grade in eleven schools. They used the interview technique and tabulated children's responses according to Buswell-John's tables of errors. From the results of this study, William and Whitaker formulated four conclusions. First, there is an irregular development in learning the fundamental processes; pupils learn certain of the arithmetic skills without mastering others. Secondly, until the particular skill causing the difficulty is designated, no progress can be realized in improving the achievement of the child. Thirdly, group testing shows that pupils are failing to secure correct results, but it does not disclose what specific skills are causing difficulty. Finally, patient, individual study of how a child moves through the complex maze of the arithmatic processes discloses the deficiencies, the partially established habits and the bad habits of the pupil.

On the other hand, a different type of diagnostic study was conducted by Ross (1964). He developed a case study description of twenty sixth and seventh grade students of average or above average intelligence and who were two years or more below their mental grade level in arithmetic achievement. The following generalizations were made in this study:

1. Arithmetic underachievement did not become apparent until the fourth grade.

2. Children tended to be from home environments which provided little intellectual stimulations.

3. Children were withdrawn and defeated in their attitude toward school.

4. Children were underachieving in school subjects other than arithmetic.

MANIPULATIVE AIDS IN ARITHMETIC

Use of Manipulative Aids

With the advent of "modern" mathematics, educators have written many articles reflecting their opinions or research regarding the use of manipulative aids in the teaching of arithmetic. Educators believe that models or manipulative aids will help children understand mathematical principles. For example, Drasin (1957) believes that arithmetic instruction should begin with concrete experiences. He equates concrete experiences with the child manipulating physical objects to arrive at a solution to a problem. Next, instruction should move to the semi-concrete level. By semi-concrete, Drasin means the level at which a quantity is shown in two dimensional representative materials. Later, the child will solve problems at the abstract level; manipulation of symbolic representations. Furthermore, he feels that instruction at the semi-concrete level: (1) acts as a bridge between concrete experiences and the abstract, (2) helps a child to concentrate upon the concept of quantity and (3) strengthens a child's understanding of mathematical processes.

Support for the use of manipulative aids was given by Hartung (1969) when he stated that the basic purpose of manipulative aids is to help children become acquainted with mathematical principles. He feels that manipulative devices should be extensively used. However, teachers should not allow themselves to become so absorbed in efforts to devise or use special instructional methods and devices that undue amounts of pupil time and energy is diverted from the main task, learning mathematics.

Some guidelines for the use of manipulative aids have been suggested by educators. For instance, Osborn (1961) suggested that models must resemble the actual physical situation so that valid conclusions can be drawn. Also, the model or the use of manipulative aids must be understood if it is to be effective.

Furthermore, Hertz (1957) believes that manipulative devices may be used in the following ways: (1) help the teacher demonstrate the meaning of a new concept, (2) used by the child as he practices for ease in computation, and (3) used by the child as he practices to gain speed and accuracy in recall. Also, more than one device should be used to introduce a new concept. since a child may understand one device more clearly than another.

Likewise, Sanders (1964) suggested four guidelines for the use of models: (1) each child should use the model at his desk to solve his problems, (2) models must be an actual application of the mathematical principle presented, (3) all prerequisite skills and concepts must be presented prior to the use of the models, and (4) each child must know how to use the model.

Moreover, Bernstein (1963) has indicated that there should be a correlation between the operations which are carried on with a device and the operations which are performed when doing the same mathematics with pencil and paper. Also, the use of manipulative aids should involve some moving parts, or should be something which is moved in the process of illustrating a mathematical principle. Equally important is the use of manipulative and pictorial materials which exploit as many senses as possible.

As an aid to educators purchasing manipulative materials, Davidson (1968) has compiled an annotated bibliography of manipulative devices and their possible sources.

Effectiveness of Manipulative Aids

Harshman (1962) conducted a study involving three treatments with a total sample of 650 children. Treatment A used commercial materials valued at $5 per child. Treatment B used less expensive materials valued at $1 per child; on the other hand Treatment C used materials selected and constructed by the teacher. The findings indicated that teacher-made materials were as effective as either expensive or inexpensive purchased materials.

In a study which apparently yielded results that contradicted these findings, Reddell and DeVault (1960) used twenty four fifth grade classes to compare three treatments involving manipulative aids. One treatment used calculators, one treatment used abacounters, and the third treatment used teacher made aids such as place value charts. At the conclusion of the study, results showed that the two commercial aids increased some aspects of achievement more than teacher made aids.

Cuisenaire materials have attracted the attention of educators within the last ten years. Hollis (1965) has reported that the Cuisenaire program taught traditional subject matter as well as the traditional method when measured by a traditional achievement test. Moreover, Nasca (1966) conducted a study whose results supported this.

Additional support for the use of manipulative aids was found by Earhart (1964) when she compared the achievement of children who only used workbooks with the achievement of children who used the abacus along with their workbooks. The experimenter found that children using the abacus obtained higher computational scores than the other children. However, Jamison (1964) conducted a similar study using a large abacus, individual abaci, or the chalkboard. No significant differences were found in this situation.

Similarly, Schott (1957) ran a study for two years involving children in grades one, two, and three. Treatment A used only the textbook, and Treatment B used manipulative devices in addition to the textbook. He found a significant difference in achievement which favored the treatment using the manipulative aids.

In a like manner, Spross (1962) studied the effect of tangible and conceptualized presentation of arithmetic to children in the fifth and sixth grades as compared to a routine classroom presentation. The experimental group used manipulative devices in developing mathematical concepts whenever possible. As contrasted to this, the control group's instructional program was structured by the arithmetic text. At the termination of the study, a significant difference favoring the experimental treatment was found.

Some studies show that children using manipulative aids are superior in their understanding of arithmetic; however, children's computational skills are not significantly increased. For example, Ekman (1967) compared the effectiveness of three ways of presenting addition and subtraction ideas to third grade pupils. Treatment I consisted of presenting the ideas immediately in algorithm form; Treatment II developed the ideas using pictures before presenting the algorithm; Treatment III developed the ideas using cardboard discs, manipulated by the pupils, before presenting the algorithm. Ekman concluded that Treatment III was superior to Treatments I and II as measured on a test of understanding. However, on a test of computational skill, there was no significant difference.

Similar results were found by Lucas (1967) when he conducted a study to determine the effect of a program of attribute block training on the multiplication of relations and the conceptualization of addition-subtraction relations. The experimental group received attribute block instruction while the control group received the Greater Cleveland Mathematics Program for first grade. The results showed that the experimental group was superior in their ability to conserve cardinality and to conceptualize addition-subtraction relations. However, the experimental group did not learn computational procedures to the same extent as children taught in a conventional program.

Nevertheless, another study yielded different results. In this study, Weber (1970) compared two methods of introducing mathematics to first grade children: manipulative versus paper and pencil. The two treatment groups were: (1) reinforcement of mathematical concepts through the use of paper and pencil follow-up activities and (2) reinforcement of mathematical concepts through the use of manipulative and concrete materials for follow-up activities. At the conclusion of the study, there was no significant difference between methods although a definite trend favored Treatment II.

Additional information justifying the use of models was found when Anderson (1957) investigated the effect of a kit of sixteen visual tactual devices upon the learning of a unit involving area, volume, and the Pythagorean Theorem. The investigation lasted eight weeks and involved 541 eighth grade students. The results of the study indicated that students using the kit scored higher but not significantly higher on a achievement test.

Considering the manipulation of visual aids from another viewpoint, Toney (1968) conducted a study which compared the achievement in basic mathematical understandings when students individually manipulated the instructional materials and when they saw a teacher demonstration of the same materials. Although no statistically significant difference was found, the results indicated a trend toward greater achievement by the group who individually manipulated materials.

REMEDIAL PROGRAMS IN ARITHMETIC

Children develop many kinds of faulty learnings and deficiencies while studying arithmetic, any one of which may contribute to a specific or general disability. Since these deficiencies will vary from child to child, it is not expected that there can be a uniform approach to the correction of learning difficulties in arithmetic. This is reflected in articles by educators expressing their opinions or reporting their research on remedial instruction.

For instance, Ross (1963) suggests that any remedial instruction program should have these general characteristics: (1) provide opportunities for the child to be successful, (2) provide immediate feedback of results, (3) provide individual attention and encouragement, (4) use concrete devices for illustrating concepts, and (5) provide adequate drill in a variety of situations.

Furthermore, Greene (1930) stated that three distinct types of drill materials must be utilized if remedial instruction is to be effective. First, initial instruction must be accompanied by adequate opportunity for repeated experience with the new concept. Secondly, periodic review for maintenance purposes. Lastly, a complete sampling of each of the basic skills that are presented for initial learning and that finally make for arithmetical achievement. He feels that drill must be constructed to fit a particular purpose. Furthermore, the distribution of practice afforded in the drills must be carefully controlled.

Educators have conducted remedial programs which employ different diagnostic techniques and instructional activities. For example, Harvey (1953) described a remedial program which essentially consisted of testing-reteaching-retesting. A diagnostic test was administered to sixth grade children. Then teachers prepared worksheets for children on the basis of their performance on the diagnostic test. These worksheets contained computational problems just like the exercises on the diagnostic test. Succeeding test scores indicated that most children improved in computational ability and eliminated many errors. However, no statistics were used to test for significance.

Many remedial programs are studied for a short period of time; however, Cotter (1967) reports a two year study involving first grade children. An experimental group of fifteen children studied the regular curriculum taught by specialized teachers in small group situations. In contrast to this, the control group of nine children participated in the regular arithmetic program. At the conclusion of the study, the mean gain in achievement for the experimental group was one year and seven months. On the other hand, the control group experienced a mean gain of only eight months. The experimenter concluded that special classes, small group and individualized instruction, and sufficient time allowance to suit individual learning rates were key factors in the successful achievement of the experimental group. Furthermore, for the experimental group, all possible senses were utilized, with much emphasis on concrete materials and illustrations.

Some educators do not feel that a small group environment is as effective as an individualized environment. For example, Bernstein (1956) found that individualizing instruction, dealing with all types of student deficiencies, was much more effective than instruction of large groups on corrective exercises. Individualized instruction accomplished more in less time than large group instruction. Gains in self confidence and increased liking for mathematics were evident in both situations, but they were more clearly evident in individualized instruction situations.

Additional information regarding an individualized remedial program was supplied when Tilton (1947) conducted a study to test the belief that a small amount of individualized instruction could produce demonstrable results. Individual instruction was given to thirty-eight fourth grade children for twenty minutes per week. Remedial instruction was based on the child's performance on a diagnostic test. In contrast to this the control group did not receive any individualized instruction. The control group's gains in achievement was subtracted from the gains made by the experimental group. After subtraction, the experimental group averaged five months more progress than the control group.

Furthermore, Chase (1929) reported a study involving seventeen junior high school students who received individually planned remedial instruction for three months. Diagnosis was by written test and interview. At the conclusion of the study, a mean gain in achievement of one to two years was common. He drew the following conclusions from the results in the study:

1. A common cause of difficulty in solving verbal problems was found to be insufficient mastery of fundamentals.

2. Lack of familiarity with combinations and processes is but one factor contributing to failure and not necessarily the most important.

3. Marked improvement can be brought about by the application of corrective treatment based upon systematic study and diagnosis of individual difficulties.

Likewise, Guiler (1929) discussed a remedial instruction program in which individual needs and difficulties dictate the course of instruction. Remedial instruction consisted in reteaching a skill if the circumstances required it and in the provision of abundant practice on the skill. Instruction was administered as individualized group instruction. To accomplish this, all pupils who encountered the same type of difficulty were grouped for purposes of instruction. Individualization was made possible by having each pupil within a group concentrate on his particular difficulties. At the end of twelve weeks of instruction, the mean gain in achievement was one year.

Less individualized remedial treatments were employed by Kathleen (1926) when she conducted a study involving twenty-six seventh graders and twenty-five sixth grade children. A diagnostic test was administered, and the errors were analyzed and classified. Then teachers of the experimental group were given a class record, class error sheet and individual error sheets. Instruction was based on this data. On the other hand, teachers of the control group received individual scores and the class median. All students drilled ten minutes a day on fundamentals for six weeks. Results of the study indicated a significant gain in achievement for the experimental group. Further, speed of performing the fundamental operations was increased for both groups.

In another study, Neal and Foster (1928) constructed diagnostic and remedial materials on fractions for fifth grade children. They analyzed the types of exercises found in textbooks on common fractions. Then a diagnostic test was formulated containing several examples of each type of exercise. Later, practice sheets to be used for remedial work were constructed to go with each type of example found on the diagnostic test. Essentially the remedial program for the experimental group was cyclic in nature: (1) diagnostic test, (2) three worksheets, (3) diagnostic test, (4) three worksheets, and (5) maintenance drill. In contrast to this, the control group used the traditional procedure for studying fractions. The experimenters concluded that organized practice materials in the hands of children with provision for diagnosis and remedial work was more effective for maintaining skills in manipulation of common fractions than the usual practice provided by teachers.

A different instructional plan was used by Otto (1927) for a remedial program. First, children were placed in different groups according to needs. Then the daily forty minute period was divided so that each pupil drilled from five to fifteen minutes on each of the processes in which he was weak. Later in the period, a practice sheet on each operation was completed. Furthermore, each child drilled on flash cards and completed a worksheet on combinations as a timed exercise. By the end of the study, children had increased in rate and accuracy. Also, children's performance on a test administered several days after this study indicated that they had retained the skills and abilities developed during the period of remedial instruction.

On the other hand, Higgins and Rusch (1965) chose seventy-eight fifth and sixth grade children to participate in a study comparing a remedial program structured by a programmed textbook with a remedial program structured by a workbook. Instruction was given for twenty minutes a day for sixteen weeks. However, there was no significant difference in achievement at the end of the instructional period.

In contrast to many remedial programs which emphasized paper-pencil drill activities, Lerch and Kelly (1966) studied the effect of a discovery and inquiry oriented remedial program upon the achievement of seventh grade students. Instruction was separated into topical units. Intra-class groupings were used to adjust content, instructional materials, and procedures to the needs of students. The experimental group used inquiry, exploration, and discovery while the control group worked in textbooks. At the end of the study there was a significant difference in achievement favoring the experimental group.

Another philosophical orientation towards remedial programs was applied by Jacqueline (1941) when she designed a program for seventh grade students. They participated in the regular mathematics class but did remedial work outside of class on a regular basis. First, a diagnostic survey test was administered. Then individualized remedial practice was provided on the basis of the child's performance on the test. Results indicated that students made remarkable gains in achievement during the study.

A case study approach for studying the effects of a remedial program was used by Cooke (1931). Six fifth grade children with learning difficulties in mathematics were selected for study. They were administered a diagnostic test. Later a written diagnostic report along with suggestions for remedial measures for each child was given to the regular fifth grade teacher. Three students were given thirty minutes of individual attention on four days per week. The other three students were treated the same as their peers in the regular arithmetic class. At the end of six weeks a second diagnosis was made. From this data, Cooke concluded that individual remedial instruction was more profitable than remedial instruction given in a group situation.

Furthermore, Fogler (1953) described a three month study involving twenty-three children. The remedial program was structured by a workbook; however, concrete, manipulative aids were extensively used to help children correctly develop concepts. Individual help was given when a child was learning a new concept or correcting a poorly developed concept. Upon termination of the study, the mean gains in achievement varied from nine months to two years, four months.

In contrast to this, Dougherty (1962) conducted a one year study involving ten fourth grade classes with median achievement below grade level. Standardized tests were administered to each child, and he was allowed to diagnose his reasons for the errors. Then teachers made an item analysis of tests so that they were aware of the errors committed by the children. Instruction was based on the class and individual needs. Timed tests were given once a month on each of the four operations. From these scores, a class chart was made to show progress in achievement. At regular intervals screening tests were given to indicate weaknesses and strengths of pupils. Also, five to ten minutes each day, children practiced the basic combinations. A resource teacher taught once a week. At the end of the study, test results indicated that all classes improved in achievement with the median achievement above grade level.

Some educators see the computer as aiding teachers in diagnosing learning difficulties and providing remedial instruction. Therefore, Shaw (1968) conducted a study to investigate the effects of three instructional strategies on arithmetic achievement of seventh grade students. The materials were remedial in nature and were presented via CAI. The instructional strategies used were drill (feedback after five problems), drill with feedback on each problem, and mixed drill. For mixed drill the subject proceeded as though it was drill strategy unless he missed two or more out of five problems. Then he was branched to drill with feedback. After completion of these items (five problems), he was branched back to the drill items at the next higher group of five problems. The order of adjusted mean scores was: mixed drill, drill with feedback, and drill.

In another type of individualized remedial program, Sherer (1968) selected forty-seven pupils in grades three through seven to participate in a tutorial remedial program. Each child was at least one year below grade level in arithmetic achievement. Fifteen students were randomly assigned to a control group which received regular classroom instruction. Experimental group I and II received twenty forty minute tutoring lessons by undergraduate elementary education majors. The tutors of experimental Treatment I utilized Brueckner's Diagnostic Tests and Self-Helps in Arithmetic. On the other hand, tutors of experimental Treatment II employed materials developed by Sherer. These materials involved instructional aids such as drawings, counters, number lines, and number charts. In addition, pupils were encouraged to formulate generalizations. At the completion of the study, no significant difference was found among the three groups in achievement.

ATTITUDE TOWARD ARITHMETIC

Some children are not successful in their study of arithmetic, and there are some aspects of arithmetic which they dislike. The attitude children have toward arithmetic is important and may influence the amount of effort expended when studying arithmetic.

Methods of Measuring Attitudes

Corcoran and Gibb (1961) described several techniques for measuring children's attitude toward arithmetic. They suggested these methods: (1) observation, (2) interviews, (3) questionnaires, (4) attitude scales, (5) sentence completion, and (6) content analysis of essays.

However, Brown and Abell (1965) found that observations by teachers were inadequate for appraising children's attitude toward arithmetic.

An objective method of measuring attitude toward arithmetic is Dutton's Scale (1968). This scale consists of fifteen statements expressing positive and negative attitudes toward arithmetic. An individual checks the statements which express his feelings toward arithmetic. Though this scale was originally constructed to measure attitudes of prospective elementary school teachers, it has been successfully administered to children as early as the third grade.

Further, Fedon (1958) used a slight variation of the Dutton Scale. Children indicated the intensity of their attitudes with a color scheme which varied from red for extreme positive attitude through yellow (neutral) to black for an extremely negative attitude. He concluded from his study: (1) color scheme was successful, (2) children expressed a definite attitude toward arithmetic as early as the third grade, and (4) children felt that various aspects of arithmetic are enjoyable and necessary but not always meaningfully significant.

A different instrument for measuring attitude was developed by Maertens (1968). He used Osgood's Semantic Differential as a model in constructing an attitude scale. Eleven bi-polar items were written for each area under consideration. Seven blanks were included with each item to give children sufficient opportunities to express strengths of feeling toward each item. This scale was successfully used with third grade children.

Formation of Attitude Toward Arithmetic

Stright (1960) reports a study of the attitudes toward arithmetic of students and teachers in grades three through six. A total sample of 1,023 students were administered a twenty-five item attitude scale. Each student agreed or disagreed with each item on the scale. After tabulating the responses on the attitude scale, she concluded: (1) elementary teachers enjoy teaching arithmetic, (2) boys and girls like arithmetic and find it useful, (3) students have developed definite attitudes by the third grade, and (4) as children grow older they develop more meaningful concepts in arithmetic.

Moreover, Dutton (1962), Smith (1964), and White (1964) conducted studies which tended to support Stright's finding regarding the formation of a definite attitude toward arithmetic by the third grade. In general, they reported that students develop their attitudes toward arithmetic throughout the school years, from second through twelfth grade, but that grades four through six were the most influential.

Relationship Between Sex and Attitude

Capps and Cox (1969) used a three part questionnaire to measure attitudes toward arithmetic of 255 children in grades four and five. Part one contained open-ended questions allowing children to designate their best and least liked subject. Part two contained questions covering issues in arithmetic of interest to children. In part three, children ranked the most commonly taught subjects in order of preference. Results of the questionnaire showed a significant difference between fourth grade boys and girls in their attitudes toward arithmetic. However, no significant difference was found between fifth grade boys and girls.

In contrast to this, Chase (1958), Curry (1963), and Dutton (1956) found middle grade boys liking arithmetic slightly better than girls. However, if the lower grades were included, Stright (1960) found elementary school girls liking arithmetic better than boys. On the other hand, Dutton (1968) found that boys and girls who had studied "modern math" were about equal in their attitude toward arithmetic.

Attitude Toward Arithmetic

In the past, it was a widely accepted notion that arithmetic was disliked by many pupils. However, numerous studies provide results which contradict this notion. For example, Inskeep (1965), Mosher (1952), and Rowland (1963) conducted studies in which children selected arithmetic as their favorite subject. In other studies, children displayed attitudes toward arithmetic which were not quite as positive. For instance, Anderson (1958) and Chase (1949) reported studies which indicated arithmetic as being above average as a preferred subject.

Relation of Attitude to Achievement

The measurement of attitudes toward arithmetic would be of less concern if attitudes were not thought to influence children's performance in arithmetic. Thus, to determine the influence of attitude on achievement, a study was conducted by Shapiro (1962) involving thirty intermediate grade children. An interview structured by 19 questions was used to measure children's attitudes toward arithmetic. Later, the children were administered the California Achievement Test. Shapiro found that those children liking arithmetic obtained significantly higher scores on the arithmetic section of the California Achievement Test.

Additional information was supplied when Bassham and Murphy (1964) investigated the relationship between children's attitude toward arithmetic and arithmetic achievement. Dutton's Attitude Scale was administered to five sixth grade classes containing 159 children. They found four times as many children with a poor attitude toward arithmetic were .65 grade below average as were .65 above average. Also, three times as many high attitude children over achieved .65 as under achieved .65 grade. These results are significant at the .02 level of significance.

Moreover, Dean (1950), Lyda (1963), and Powell (1961) conducted studies which supported the positive relationship found by Shapiro; and Bassham, Murphy, and Murphy. Likewise, Greenblatt (1962) found that girls with high arithmetic achievement had more positive attitudes. Furthermore, Rowland and Inskeep (1963) reported a study showing that a feeling of success increased children's preference and attitudes for arithmetic. However, Anttonen (1968) was not able to find a significant relationship.

Instructional Procedures and Attitude

Certain educators believe that teachers should employ instructional procedures and activities which foster the development of positive attitudes toward arithemtic.

Thus, Lyda and Morse (1963) investigated the relationship between attitude and teaching methods. Fourth grade children received "meaningful" teaching for twenty-one periods. Their instructional procedures emphasized the mathematical aims of arithmetic: (1) stressing the concept of number, (2) understanding the numeration system and place value, (3) the use of fundamental operations, (4) rationale of computational algorithms, and (5) relationships which make arithmetic a system of thinking. Dutton's Attitude Scale was used to measure children's attitudes toward arithmetic. The results indicated: (1) when meaningful methods of teaching were used, children's attitudes toward arithmetic became more positive and (2) associated with meaningful methods of teaching and changes in attitudes were significant gains in arithmetic achievement.

Likewise, Dutton (1956), Malone and Freel (1954), and Stokes (1958) conducted studies yielding results which supported these conclusions. Similarly, Fedon (1958) found a positive increase in attitude when problem solving was related to experience.

CHAPTER III

THE DESIGN OF THE STUDY

This experiment was designed to determine the effect of diagnostic and remedial programs upon the arithmetic achievement and attitude development of fourth grade children with learning difficulties in arithmetic. Two experimental treatments were administered to determine their effects upon children's achievement in tests of: (1) knowledge of arithmetic concepts, (2) computational skills, and (3) problem solving ability. Further, the effect of these two experimental treatments upon children's attitudes toward: (1) arithmetic, (2) school, (3) reading, (4) spelling, and (5) science was studied.

A description of the two experimental treatments follows:

1. Textbook Treatment - Children assigned to this program received paper-pencil instructional activities from an arithmetic textbook whose grade level corresponded to the grade-equivalent score on a standardized diagnostic test.

2. Laboratory Treatment - Children assigned to this program received teacher-designed instructional activities involving extensive use of physical models and manipulative aids as well as written exercises. These instructional activities were developed for each skill which the child was unable to successfully perform on a diagnostic test designed by the experimenter.

The design and procedures used will be presented in the following sequence:

1. Setting of the study
2. Development of a model for diagnosis
3. Experimental units
4. Experimental treatments
5. Experimental design
6. Hypotheses to be tested
7. Evaluation instruments
8. Statistical analyses
9. Limitations of the study

THE SETTING OF THE STUDY

Since the teachers participating in the Experienced Teacher Fellowship Program at the University of Oregon were to administer the treatments, it was necessary to use the elementary schools to which these teachers had been assigned. Originally there were fifteen of these schools participating in the program: (1) two schools in the Bethel School District, (2) two schools in the Springfield School District, and (3) eleven schools in the Eugene School District. One of the schools in Springfield was eliminated from the study because of administrative and scheduling difficulties which made it impossible for the teachers to follow the treatment schedule required in the experiment. Furthermore, two schools in the Eugene School District were eliminated from the study. The size of one school was so small that the number of fourth grade children with arithmetic learning difficulties was less than the number required by the design of the experiment. Another school was eliminated from the experiment because administrative arrangements required by the experiment could not be met. Therefore, from initiation to termination of the study, twelve schools were involved: (1) one in the Springfield School District, (2) two in the Bethel School District, and (3) nine in the Eugene School District.

These schools were selected by administrators in each of the three districts in conjunction with the directors of the Experienced Teacher Fellowship Program. Criteria for selection were essentially: (1) space and facilities for two Fellows working with two small groups of children, (2) cooperativeness of principal and staff, and (3) sufficient number of children at all grade levels with learning difficulties in reading, arithmetic, and language arts. The staff within each school determined the number of children with learning difficulties from their performance on achievement tests, such as the Stanford Achievement Test or California Achievement Test.

Some socio-economic characteristics of children in each of the twelve schools involved in the experiment are given in Table 3.01. Most of the schools are in older residential districts inhabited by skilled and unskilled workers whose income is average to below average, although a few children reside in a rural setting.

Only one school has an attendance area composed of children whose parents primarily engage in professional occupations. The cost of most houses in the attendance area encompassed by the twelve schools is low to moderate.

TABLE 3.01

SOCIO-ECONOMIC CHARACTERISTICS OF CHILDREN IN THE TWELVE SCHOOLS

	Location				Houses						Income					Occupation of Parents			
					Cost			Age											
School	Business District	Residential	Sub-divisions	Rural	Low	Average	Above Average	Old - 16 years	Average - 8-16 years	New - 0-8 years	Low	Low-Average	Average	Above Average	High	Unskilled	Skilled	Professional	Agriculture
1				X		X			X			X				X			X
2				X		X			X				X			X			X
3			X			X				X			X			X	X		
4		X				X			X				X			X	X		
5				X		X				X			X			X	X		
6		X				X		X					X			X	X		
7		X			X			X				X				X	X		
8	X	X			X			X			X					X	X		
9			X				X			X				X				X	
10		X				X			X				X			X	X		
11	X				X			X			X					X			
12		X				X			X				X			X	X		

THE TEACHERS

Twenty-four teachers participated in the experiment. Twelve teachers administered the Laboratory Treatment and a similar number administered the Textbook Treatment. Two teachers were assigned to each of the twelve schools participating in the Experienced Teacher Fellowship Program. This enabled both treatments to be administered simultaneously within each school. All of the teachers were Fellows in the Experienced Teacher Fellowship Program in Learning and Behavior Disorders.

The Experienced Teacher Fellowship Program (ExTFP) is designed to further train elementary teachers of grades 1 through 6 in the State of Oregon. The program is funded by the U. S. Office of Education and conducted at the University of Oregon. As stated in the brochure describing the program, the major purpose is to provide an opportunity for the participants, regular elementary school teachers, to gain more understanding of the theories, methods and techniques developed for use in special and remedial settings and to develop the requisite skills necessary to utilize these techniques in their own classrooms and schools (Hotchkiss, 1969).

With regards to this experiment, three very pertinent objectives of the ExTFP are stated in the literature describing the program:

1. Acquaint teachers with explanatory systems that pertain to the body of knowledge dealing with child development, behavior disturbance, and remedial-learning problems.

2. Provide factual information and understanding of specific treatment strategies designed to cope with behavior problems.

3. Provide supervised field experiences in which teachers can develop skill in isolating remedial-behavioral problems and bring behavioral modification and remediation techniques to bear in treating such problems within the classroom (Hotchkiss, 1969).

For a teacher to be accepted as a participant in the Experienced Teacher Fellowship Program, the following criteria should be met:

1. The applicant should be an experienced elementary school teacher who wishes to teach in grades one through six.

2. The applicant must have engaged in elementary education for at least one full year immediately preceding the year for which the award was made.

3. The applicant must be eligible for admission to the

University of Oregon Graduate School.

4. Recommendations from the local school district where the applicant is employed and to which he plans to return.

5. The professional educational background of the applicant is considered as the program is specifically designed for the non-specialist regular elementary school teacher with little or no training in special or remedial education who wishes to return to the classroom (Hotchkiss, 1969).

The course of study for participants in the Experienced Teacher Fellowship Program is listed in Table 3.02.

Information about each teacher who participated in the experiment was obtained from application forms and transcripts which were received by the directors, Dr. James Hotchkiss and Dr. Carl Wallen, when the teachers made application to the Experienced Teacher Fellowship Program. This information is summarized in Tables 3.03 and 3.04.

Of the teachers involved in the experiment, fifteen were women and nine were men. The number of years of teaching experience for the teachers ranged from a high of twenty-nine years to a low of one year. These years of teaching experience do not include the year they were involved in the Experienced Teacher Fellowship Program, but were years of experience accumulated prior to their acceptance into the Experienced Teacher Fellowship Program. The twenty-four teachers participating in the experiment have a mean of 8.7 years of teaching experience in the elementary school.

Table 3.04 indicates the mathematics preparation at the high school and college level. In the same table, information is given regarding the participating teachers' preparation in mathematics methods courses. Courses taken during their studies in the Experienced Teacher Fellowship Program are not included.

A majority of teachers in the experiment have completed courses in algebra and geometry in high school. The mean number of units of mathematics taken in high school is 2.2 units. At the college level, several teachers had mathematics courses designed for elementary teachers. These courses stressed the essentials or foundations of mathematics. While five teachers had large numbers of credits in mathematics, only one teacher had what might be considered a strong background in mathematics appropriate for an elementary school teacher. This teacher's mathematics background included college algebra and analytic geometry and calculus. The mathematics background of the other four teachers included several credits in essentials of mathematics and introductory college algebra. The mean number of credits in college mathematics for the twenty-four teachers in the experiment was 6.7 quarter hours credit. The majority of the teachers in the experiment have taken only one mathematics methods course in which they learned to teach mathematics in the elementary school.

TABLE 3.02

CURRICULUM FOR EXPERIENCED TEACHER FELLOWSHIP PROGRAM

Summer:
- Seminar: Life Styles of the Poor
- Practicum: Alienated Youth
- Workshop: Socially and Culturally Deprived
- September Experience
- Seminar: Teaching Emotionally Disturbed
- Workshop: Flanders Interaction Analysis
- Diagnosis of Basic Skills
- Practicum: Remedial Clinic Diagnosis

Fall:
- Interpersonal Communications Theory
- Practicum: Alienated Youth
- Practicum: Programming Learning Environments
- Language and Reading for Exceptional Child
- Principles of Child Learning

Winter:
- Seminar: Planned Changes in Schools
- Practicum: Alienated Youth
- Field Experience in Behavior and Learning Problems
- Diagnosis in Teaching Language and Mathematics
- Abnormal Psychology

Spring:
- Seminar: Facilitating Grade and Systems Change
- Practicum: Alienated Youth
- Field Experience in Behavior and Learning Problems
- Seminar: Diagnosis in Mathematics
- Practicum: Remedial Math
- Direct Instruction

Electives:
- Maladjusted Child
- Psychology of Exceptional Child
- Behavior - Special Education
- Issues in Extensive Learning
- Theory of Personality

TABLE 3.03

SEX AND TEACHING EXPERIENCE OF EXPERIENCED TEACHER FELLOWSHIP PROGRAM FELLOWS

Name	Sex	Age	Yrs. Exp.	Gr. Level Taught
A	F	28	7	1 - 3
B	F	46	4 1/2	1 - 2
C	M	28	5	4 - 6
D	F	25	5	3 - 4
E	F	30	8	2 - 4
F	F	35	8 1/2	1 - 3
G	M	36	7 1/2	1 - 6
H	F	37	8	4 - 5
I	F	43	4	mentally retarded
J	F	38	11	1 - 6
K	F	32	11	1, 4, 6
L	M	34	6	6
M	M	35	10	5 - 6
N	M	25	3	5
O	M	34	11	4 - 7
P	M	51	13	1 - 8
Q	M	34	13	6
R	F	30	5	6
S	F	35	15	1 - 2
T	F	55	29	*N - 3
U	F	27	1	2
V	M	27	3	3 - 4
W	F	31	4	4
X	F	29	7	1 - 2
Mean		37.37	8.7	

*Nursery School

TABLE 3.04

UNITS IN MATHEMATICS AND MATHEMATICS METHODS COMPLETED BY EXPERIENCED TEACHER FELLOWSHIP PROGRAM FELLOWS

Name	H.S. Math	College Math	Math Methods
A	2	3	3
B	1	3	6
C	1.5	19	3
D	3	3	3
E	3	14.5	5
F	2	2	3
G	1.5	6	6
H	2	4.5	4.5
I	1	4	0
J	2	3	3
K	2	3	5
L	2	20	9
M	2	3	3
N	2	3	2
O	3	4.5	3
P	-	4.5	3
Q	2	0	4.5
R	3	6	0
S	3	6	0
T	4	17	0
U	1	6	3
V	4	17	3
W	2.5	3	0
X	2	6	5
Mean	2.2	6.7	3.2

DEVELOPMENT OF A MODEL FOR DIAGNOSIS

This experiment involved two diagnostic and remedial instructional programs for fourth grade children who have learning difficulties with addition, subtraction, multiplication and division of whole numbers.

For the Textbook Treatment, the diagnostic procedures consisted of the administration of a standardized diagnostic arithmetic test. Then the teacher obtained from the accompanying test manual a grade equivalent score corresponding to the child's raw score on the diagnostic arithmetic test. Later, for each child, the teacher chose an arithmetic textbook whose grade level corresponded to the child's grade equivalent score.

However, for the Laboratory Treatment, the experimenter developed a model for diagnostic procedures. This diagnostic model consisted of: (1) hierarchies of subordinate arithmetic skills, (2) behavioral objectives defining each skill in the hierarchies, (3) four levels of testing and, (4) diagnostic arithmetic tests containing exercises designed to measure a child's knowledge of each skill in the hierarchies.

A hierarchy of subordinate arithmetic skills was derived from the task analysis of the four mathematical operations on whole numbers. Initially a terminal goal was chosen. For this experiment, the terminal goal was:

> "A child will successfully apply a division algorithm to relevant data in a problematic situation requiring the use of the division operation."

At this point the following question was asked:

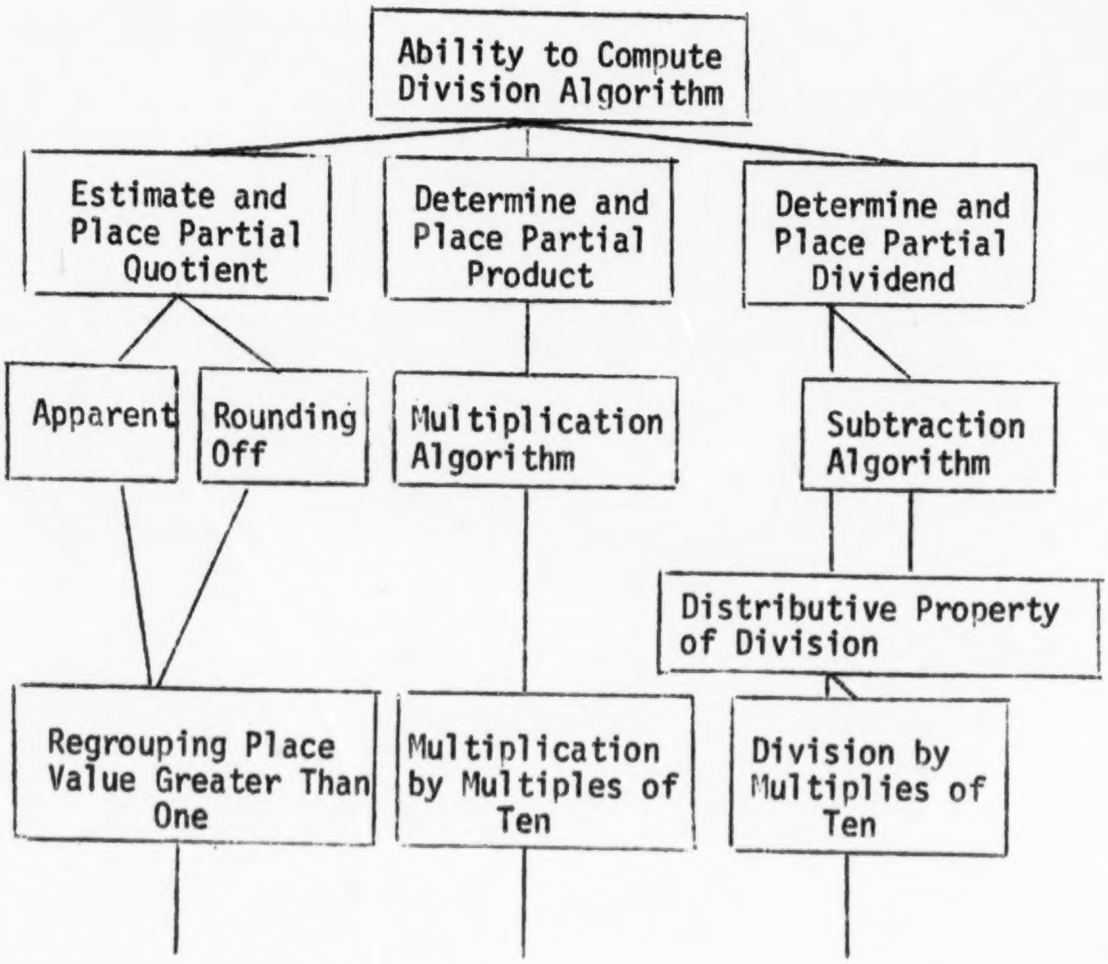
> "What subordinate arithmetic skills must the child possess to be able to successfully perform this task?"

In response to this question, certain lower level division skills were identified: (1) estimate and place partial quotient, (2) determine and place partial product, and (3) determine and place partial dividend. Refer to the hierarchical model in Table 3.05 to determine the placement of these subordinate skills in relationship to the terminal objective.

Then the same question was asked regarding each skill that was identified. Additional skills were identified and placed on a level below the previously identified skills. Thus, successive levels of subordinate division skills were identified. Eventually subordinate skills involving the other three operations were identified

TABLE 3.05

MODEL OF A DIVISION HIERARCHY


Ability to Compute Division Algorithm
Estimate and Place Partial Quotient
Determine and Place Partial Product
Determine and Place Partial Dividend
Apparent
Rounding Off
Multiplication Algorithm
Subtraction Algorithm
Distributive Property of Division
Regrouping Place Value Greater Than One
Multiplication by Multiples of Ten
Division by Multiplies of Ten

and included in the hierarchy. This process was continued until subordinate skill requiring the discrimination of physical characteristics was reached. Hierarchies for each of the mathematical operations and a composite hierarchy are found in Appendix A.

Next, each skill in the hierarchies was defined in terms of the behavior that a child must exhibit when he can successfully perform the skill. This was done using Mager's (1962) guidelines for writing behavioral objectives. These behavioral objectives include three distinct parts: (1) a description of the terminal behavior of the child, (2) conditions under which the child is to exhibit the terminal behavior, and (3) criteria for judging when the child can successfully perform the terminal behavior.

Behavioral objectives used in this experiment for each skill in the hierarchy of subordinate arithmetic skills are in Appendix B. These behavioral objectives guided the experimenter in the formation of the diagnostic test which was used in the Laboratory Treatment for diagnosing a child's learning difficulties in arithmetic. Also, the behavioral objective for a particular skill aided the teachers of the Laboratory Treatment when they designed instructional activities and decided whether the child could successfully perform the skill.

A diagnostic arithmetic test was constructed by the experimenter to assess whether a child could successfully perform each skill in the hierarchy of subordinate arithmetic skills. Whenever possible, the test questions required the child: (1) to manipulate objects illustrating a mathematical principle, (2) to write a mathematical sentence describing the action performed on physical objects by the examiner, and (3) to use paper-pencil for computing the answer to a problem. This evaluative instrument is described in further detail in the "Evaluation Instruments" section of this chapter.

EXPERIMENTAL UNITS

By the fourth grade, some children have developed learning difficulties with addition and subtraction of whole numbers, since they have studied these two operations for the greater portion of three school years. Furthermore, many of these children have learning difficulties with multiplication and division of whole numbers. It was because of these reasons that the experimenter chose fourth grade children to participate in the experiment.

At the beginning of the experiment, 74 children were in the Textbook Treatment and 73 children were in the Laboratory Treatment. When final testing was completed on June 5, 1970, there were 71 children in the Textbook Treatment and 72 children in the Laboratory Treatment. Children absent from schools on the days of final testing and children who moved from the districts accounted for the difference in the number of children from the beginning to the completion of the experiment.

Tables 3.06 through 3.09 contain mean achievement scores for children in the Laboratory Treatment and Textbook Treatment at the beginning of the study. Furthermore, Table 3.09 contains grade equivalent scores for the total arithmetic test. Mean achievement scores are reported for the total arithmetic test battery and its three subsections. Grade equivalent scores are shown for the total arithemtic test battery and were obtained from a norm table given in the examiner's manual which accompanies the Comprehensive Test of Basic Skills: Arithmetic. This test was administered at the beginning of the eighth month of the 1969-70 school year; therefore, the average grade equivalent score for the children should be 4.8. As can be seen from the tables, children from every school except School 8 in the Textbook Treatment were at least one grade equivalent below the average. Children's grade equivalent scores for the Textbook Treatment ranged from a low of 2.8 to a high of 4.1 while the scores of children in the Laboratory Treatment ranged from 2.8 to 3.7.

With the exception of School 4, class size in both treatments was limited to five or six children each. In School 4, students with learning difficulties in mathematics were ability grouped prior to the experiment. So the twenty children in the lowest achieving group were randomly assigned to the Laboratory Treatment and the Textbook Treatment - an assignment of ten children for each treatment.

TABLE 3.06

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE COMPUTATIONAL SKILLS SUBTEST OF THE COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM Q1

School	Laboratory Treatment	Textbook Treatment
1	41.00	40.80
2	23.67	23.33
3	28.00	34.83
4	40.90	40.10
5	33.33	37.33
6	32.00	36.40
7	42.20	31.33
8	36.80	49.00
9	37.83	33.50
10	34.67	42.50
11	32.67	28.67
12	40.50	42.00
TOTAL	35.41	36.71

TABLE 3.07

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE KNOWLEDGE OF CONCEPTS SUBTEST OF THE COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM Q1

School	Laboratory Treatment	Textbook Treatment
1	15.80	13.20
2	10.17	12.00
3	9.67	15.50
4	18.10	20.80
5	19.50	17.17
6	14.17	17.80
7	17.80	15.50
8	15.60	19.67
9	20.00	18.00
10	12.83	18.33
11	13.17	12.83
12	18.33	18.00
TOTAL	15.53	16.81

TABLE 3.08

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE APPLICATIONS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM Q1

School	Laboratory Treatment	Textbook Treatment
1	9.40	7.60
2	9.00	7.17
3	7.17	10.83
4	11.80	14.30
5	14.50	11.33
6	6.00	9.60
7	10.60	9.17
8	11.80	13.17
9	13.33	11.00
10	10.33	9.67
11	9.50	8.17
12	9.67	11.60
TOTAL	10.33	10.55

TABLE 3.09

MEAN SCORES AND GRADE EQUIVALENT BY SCHOOL AND TREATMENT FOR THE COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM Q1

	Laboratory Treatment		Textbook Treatment	
School	Mean Score	Grade Equivalent	Mean Score	Grade Equivalent
1	66.20	3.5	61.60	3.4
2	42.83	2.8	42.50	2.8
3	44.83	2.9	61.17	3.4
4	70.80	3.7	75.20	3.8
5	67.33	3.5	65.83	3.5
6	52.17	3.1	63.80	3.4
7	70.60	3.7	56.00	3.2
8	64.20	3.4	81.83	4.1
9	71.17	3.7	62.50	3.4
10	57.83	3.3	70.50	3.7
11	55.33	3.2	49.67	3.0
12	68.50	3.6	71.60	3.7
TOTAL	61.27	3.4	64.07	3.4

EXPERIMENTAL TREATMENT

ADMINISTRATION OF THE TREATMENTS

Laboratory Treatment and Textbook Treatment were administered by teachers participating in the Experienced Teacher Fellowship Program at the University of Oregon. During the winter term of the 1969-1970 academic year all of the teachers met with Dr. Norbert Maertens and the experimenter for three hours each week from January 7 to March 16, 1970, a total of thirty-three hours of instruction. Their instruction included:

1. Constructing a hierarchy of mathematical skills
2. Writing behavioral objectives
3. Learning and testing levels of mathematical learning
4. Constructing and administrating a diagnostic mathematics test
5. Designing and teaching instructional activities to eliminate learning difficulties in mathematics

Teachers received copies of the hierarchies of mathematics skills for addition, subtraction, multiplication and division as well as behavioral objectives defining each skill in the hierarchies to guide them while administering the treatments. A copy of the hierarchies and behavioral objectives may be found in Appendix A and B, respectively.

The Experienced Teacher Fellowship Program teachers were randomly assigned to each of the fifteen schools. Within each of the twelve schools involved in the experiment, teachers randomly assigned themselves to treatments by a flip of a coin.

During the course of the experiment, the experimenter, Dr. Maertens and three supervisors employed by the Experienced Teacher Fellowship Program visited the classes weekly to insure that the conditions required by the experimental design and treatments were properly administered. Any areas of confusion concerning the administration of the treatments were readily resolved.

During the spring term of the 1969-70 academic year, all of the teachers met with Dr. Maertens and the experimenter at the District 4J Mathematics Resource Center. Teachers from six schools met on alternate weeks for three hours of instruction per week. During this instructional period, teachers constructed physical models and games, designed instructional activities, and received additional ideas for activities from Dr. Maertens and the experimenter. Teachers administering the Laboratory Treatment were encouraged to use these models, games, or instructional activities with their children. Each group of teachers received fifteen hours of instruction from March 30 to June 5, 1970.

At the initiation of the experiment, three tests were administered by the teachers to all children involved in the experiment: (1) Comprehensive Test of Basic Skills: Arithmetic, Form Q1, (2) Stanford Diagnostic Arithmetic Test, and (3) experimenter-designed diagnostic arithmetic test. Furthermore, at the conclusion of the experiment, two achievement tests and attitude scales were administered by the teachers to all children involved in the experiment: (1) Comprehensive Test of Basic Skills: Arithmetic, Form R1, (2) experimenter-designed achievement test, and (3) attitude scales for arithmetic, school, reading, spelling, and science. All of the tests with the exception of the attitude scales were hand scored by the teachers. All attitude scales were scored by the experimenter.

Teachers were required to keep a case study file for each child involved in the experiment. In the case study file for each child were found these items:

1. Comprehensive Test of Basic Skills: Arithmetic, Forms Q1 and R1.
2. Stanford Diagnostic Arithmetic Test
3. Experimenter-designed diagnostic arithmetic test
4. Experimenter-designed arithmetic achievement test
5. Attitude scales for arithmetic, school, reading, spelling, and science
6. Case study forms
7. Instructional activities forms

The instructional activities forms contain information regarding the activities which were performed by the child. These forms enabled the experimenter to document the administration of the two experimental treatments. At the conclusion of the experiment, these case study files were given to the experimenter.

NATURE OF THE TREATMENTS

The experiment was designed to measure the effects of two diagnostic and remedial programs upon children's arithmetic performance and to measure the effects of these two programs on the attitudes of children in five different areas. The three areas of arithmetic achievement studied were: (1) computational skills, (2) concepts, and (3) applications of mathematical processes. Areas selected for measuring attitudes were: (1) arithmetic, (2) school, (3) reading, (4) spelling, and (5) science.

In each of the twelve schools involved in the experiment, the two Experienced Teacher Fellowship Program Fellows met with the fourth grade teachers to identify twelve children who were having learning difficulties in arithmetic. Criteria used for deciding whether a child had learning difficulties in arithmetic were: (1) performance on the arithmetic test of an achievement battery administered throughout the district, (2) performance on teacher-constructed tests, (3) performance on daily written assignments and (4) performance on daily instructional activities. For the child to be included in the experiment, his performance on the Comprehensive Test of Basic Skills: Arithmetic Test, Form 01 must be below grade level. Also, the child's performance on teacher-constructued arithmetic tests must have been below the average for the total class.

Once the twelve children were selected, their names were written on individual slips of paper and placed in a box. One name was drawn at a time to assign children to the two treatments. Thus, within each school, initially six children participated in each treatment. Later some children were dropped from the experiment because they moved or were absent on testing days. However, School 4 had ten in each treatment group because the fourth grade children had been ability grouped prior to the experiment; therefore, the twenty children placed in the lowest ability group were selected to participate in the study.

Tables 3.12 and 3.13 show the initial and final testing days for the experiment.

During the instructional phase, children participating in the experiment received instruction from the Experienced Teacher Fellowship Program Fellows instead of their regular classroom teacher. Furthermore, these children did not participate in the regularly scheduled arithmetic class. Instead, instruction was given for forty-five minutes per day in separate rooms from the rest of the fourth grade children.

Table 3.10 shows the number of days that children in each school were instructed. Instruction began on April 13, 1970 and ended on May 27, 1970. Essentially, there was a seven week period of instruction during the experiment.

Due to some committments of the Experienced Teacher Fellowship Program Fellows to their program, it was necessary for one or both Fellows to be gone from their school on certain Fridays. If this was the case, the remaining Fellow would teach both treatment groups. The instructional activities performed by children on these days involved the study of geometry or fractions and did not involve any work with the four operations on whole numbers. If both Fellows were required to be away from their school, the children in both treatment groups remained in the regular arithmetic classes which, at this time of year, were studying fractions or geometry.

Neither of these days nor the days of testing were included in the number of days shown in Table 3.10.

TABLE 3.10

NUMBER OF DAYS OF INSTRUCTION

School	Number of days
1	29
2	28
3	30
4	28
5	28
6	28
7	28
8	30
9	30
10	29
11	29
12	28
Mean	28.7

LABORATORY TREATMENT

The diagnosis of a child's learning difficulties in arithmetic was based on the child's performance on the experimenter-designed diagnostic arithmetic test. This test was administered individually or to a small group of two or three children. From the child's performance on this test, skills were identified in the hierarchy which the child could not successfully perform. Then these skills were marked on the hierarchy of arithmetic skills.

The remedial instructional program began with the skill at the lowest level in the hierarchy which the child could not successfully perform. When the child exhibited the behavior specified in the behavioral objective for this skill, he performed instructional activities involving skills at successively higher levels in the hierarchy which he could not successfully perform on the diagnostic test. Thus, the child's rate of progress through the hierarchy was determined by the degree of success which the child achieved while performing the instructional activities designed for a particular skill.

Teachers designed all instructional activities for skills in the hierarchies which a child could not successfully perform. Furthermore, arithmetic textbooks were not to be used with children in this treatment. Moreover, the instructional activities were to make use of physical models, manipulative aids, games, patterns, and paper-pencil exercises. Teachers used physical models and aids as they illustrated mathematical concepts. However, what is more important, is that the children manipulated the physical objects and aids while the teacher manipulated objects or when they were solving exercises.

Games were designed to illustrate mathematical concepts and to help children improve their computational skills. Patterns in arithmetic were constantly emphasized by the teacher through instructional activities which the children were to perform.

TEXTBOOK TREATMENT

For this treatment, the diagnosis of a child's learning difficulties in arithmetic was based upon the child's performance on the Stanford Diagnostic Arithmetic Test. Each child was administered subtests: (1) Operations - 1B, (2) Place value - 1C, (3) Addition - 2A, (4) Subtraction - 2B, (5) Multiplication - 2C, and (6) Division - 2D. The sum of the raw scores from subtests 2A, 2B, 2C, and 2D was converted to a grade equivalent score for each child in the treatment.

Then, an arithmetic textbook whose designated grade level corresponded to the child's grade equivalent score was chosen by the teacher. The fractional part of the equivalent score was used to estimate the place in the textbook for the child to begin the remedial program.

From this initial point, the child moved from page to page performing the instructional activities on each page. Each child's rate of progress was determined by the speed at which he could successfully perform the instructional activities.

In essence, the remedial program consisted of the child performing the instructional activities which were suggested in the textbook that was given to him. Furthermore, the teacher could use only the instructional suggestions that appeared in the teacher's guide accompanying each arithmetic textbook which she chose. In most situations, these instructions suggestions involved paper-pencil exercises, black-board exercises, or teacher-directed verbal exercises. Moreover, any manipulative aids suggested in the teacher's guide could be used by the teacher but not by the children. Only ditto sheets of exercises similar to those in the textbook and its accompanying teacher's guide could be used to supplement instruction by the teacher or textbook.

Textbooks chosen by teachers for use in the study are shown in Table 3.11.

TABLE 3.11

TEXTBOOKS USED BY CHILDREN IN THE TEXTBOOK TREATMENT

School	Name	Publisher	Edition	Grade Level
1	Seeing Through Arithmetic	Scott, Foresman, & Co.	1963	3
2	Greater Cleveland Math Prog	SRA	1962	1, 2, 3
3	Arithmetic	Laidlaw Brothers	1965	3, 4
4	Elem School Mathematics	Addison Wesley	1964	3, 4
5	Greater Cleveland Math Prog	SRA	1962	3
6	Seeing Through Arithmetic	Scott, Foresman & Co.	1963	3, 4
7	Greater Cleveland Math Prog	SRA	1962	3
8	Greater Cleveland Math Prog	SRA	1962, 1964	3, 4
9	Greater Cleveland Math Prog	SRA	1962	3
10	Seeing Through Arithmetic	Scott, Foresman & Co.	1963	3
	Elem School Mathematics	Addison Wesley	1962	2
	Exploring Arithmetic	Webster Publ. Co.	1957	4
11	Exploring Arithmetic	Webster Publ. Co.	1957	3
12	Greater Cleveland Math Prog	SRA	1962, 1964	3, 4

EXPERIMENTAL DESIGN

Fourth grade children in twelve schools within the Greater Eugene Metropolitan Area were involved in this experiment. Two Experienced Teacher Fellowship Program Fellows were randomly assigned to each school. Furthermore, within each school, the Fellows randomly assigned themselves by a flip of a coin to the two treatments which were conducted simultaneously.

Within each school, twelve fourth grade children were selected by regular classroom teachers and the two Fellows. However, twenty students were selected in one school because of ability grouping policies. Regardless of the number of students chosen, students were selected who have learning difficulties in arithmetic as evidenced by their performance on daily arithmetic assignments, teacher-constructed arithmetic tests, and standardized arithmetic achievement tests. Moreover, Fellows randomly assigned students to treatments by drawing students' names from a box.

At the beginning of the study, three tests were administered by the Fellows to all children participating in the study: (1) Comprehensive Test of Basic Skills: Arithmetic, Form Q1, (2) Stanford Diagnostic Arithmetic Test, and (3) experimenter-designed diagnostic test. A child's performance on the diagnostic test served as a basis for designing remedial programs by the Fellows. Analysis of variance procedures were applied to the results of the three tests to further assure the experimenter that no significant differences existed among the children assigned to the two treatments in their knowledge of arithmetic at the initiation of the study. Furthermore, results from the commercial arithmetic achievement test assured the experimenter that the selected children were having learning difficulties in arithmetic.

At the conclusion of the experiment, tests were administered by Fellows to all children in the study: (1) attitude scales, (2) Comprehensive Test of Basic Skills: Arithmetic, Form R1, and (3) experimenter-designed arithmetic achievement test. Two-way analysis of variance techniques were applied to the results of these tests and scales. A fixed effects model with unequal cell frequencies was used. Unequal cell frequencies occurred for reasons unrelated to the nature of the treatments used in the study: (1) children were absent on the days the tests were administered or (2) children moved to a residence outside of the school districts participating in the study.

Since the number of observations within cells were not proportional and the unequal cell frequencies were not related to the nature of the treatments, an unweighted means analysis was applicable. Two sources of variation were studied - treatments and schools. Furthermore, children were chosen as the experimental unit.

THE HYPOTHESES

The purpose of this study was to determine the effects of two diagnostic and remedial programs upon the arithmetic achievement and attitude development of fourth grade children with learning difficulties in arithmetic. Two experimental treatments were administered to determine their effects on children's achievement in tests of: (1) computational skills, (2) knowledge of arithmetic concepts, and (3) applications. Further, the effects of these two treatments upon children's attitudes toward arithmetic, school, reading, spelling, and science were studied.

Thirteen null hypotheses to be tested for the effects of the treatments were established. They are as follows:

1. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

2. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the mathematical concepts test of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

3. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the applications test of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

4. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on total battery of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

5. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the manipulation test of the experimenter-designed achievement test.

6. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the experimenter designed achievement test.

7. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the applications test of the experimenter-designed achievement test.

8. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the total battery of the experimenter-designed achievement test.

9. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitudes toward arithmetic.

10. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitudes toward school.

11. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitudes toward reading.

12. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitudes toward spelling.

13. No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitudes toward science.

THE EVALUATIVE INSTRUMENTS

Due to the nature of the treatments and purpose of the experiment, two standardized achievement tests, one experimenter-designed achievement test, one standardized diagnostic test, one experimenter-designed diagnostic test, and experimenter-designed attitude scales were used.

Standardized Achievement Tests

The Comprehensive Test of Basic Skills: Arithmetic, Forms Q1 and R1 were used to measure achievement at the beginning and end of the experiment, respectively. These two forms were designed to measure achievement in arithmetic computation, concepts and applications of children in grades 2.5 through 4. The test was developed by the California Test Bureau, a Division of McGraw Hill Book Company, and has a 1968 copyright date.

The computational subtest contains sixty items, fifteen items for measuring each of the four operations on whole numbers: addition, subtraction, multiplication and division. This subtest was divided into four separate parts, one for each operation, so that the children would have an opportunity to work problems in each operation. Within each section, the trend in difficulty was from less difficult items to more difficult.

The concept subtest has thirty items that measure the child's ability to: (1) recognize or apply the appropriate operation or principle, (2) convert concepts expressed in one form to another form, and (3) comprehend numerical concepts and understand their interrelationships.

The applications subtest contains twenty items which emphasize problem solving. Test items measure a child's ability to: (1) comprehend the problem, (2) select the appropriate method, (3) organize the relevant data, and (4) solve the problem.

Standardized Diagnostic Arithmetic Test

Stanford Diagnostic Arithmetic Test, Level 1, Form W, was administered to all children involved in the study. Only teachers of the Textbook Treatment used the results of this test for diagnostic purposes. Leslie Beatty, Richard Madden, and Eric Gardner developed the diagnostic test. It was published by Harcourt, Brace, and World, Inc. and has a 1966 copyright. Level 1 was designed to be used with children in grades 2.5 through 4.

Only the following parts of this test were administered to the children in the study:

1. Subtest 1B - Operations
2. Subtest 1C - Decimal Place Value
3. Subtest 2A - Addition Computation
4. Subtest 2B - Subtraction Computation
5. Subtest 2C - Multiplication Computation
6. Subtest 2D - Division Computation

Subtest 1B, operations, has twenty-six items that measure a child's ability to use properties of the whole numbers in the four operations. Subtest 1C, decimal place value, contains thirty items to measure a child's ability to apply the principles of decimal place value numeration in reading, interpreting, and writing numerals. Subtests

2A, 2B, 2C, and 2D contain eighteen problems in each subtest. Each subtest contains problems with and without regrouping, with and without zero, and zero in different positions. Furthermore, the test consists of paper and pencil exercises and exercises dictated to the children by the teacher.

Experimenter-Designed Tests

Four levels of testing were used by the experimenter while constructing diagnostic and achievement tests for this study: (1) rote or associative, (2) concept, (3) principle, and (4) problem solving. Exercises were constructed which measured a child's knowledge of arithmetic on each of these levels.

First, the rote or associative learning involves the presentation of a stimulus to which a student gives the response that he has learned to associate with that particular stimulus. In most instances, the student does not know why his response was correct. The response may result from the student mechanically or rotely following a memorized procedure. Examples of exercises measuring a child's arithmetic knowledge at this level are the following:

"Compute the answer to each exercise."

$$\begin{array}{r} 35 \\ +\ 24 \\ \hline \end{array} \qquad \begin{array}{r} 9876 \\ -\ 5431 \\ \hline \end{array} \qquad \begin{array}{r} 142 \\ \times\ 35 \\ \hline \end{array} \qquad 24\overline{)3567}$$

Secondly, at the concept level of learning, a child is able to: (1) classify events and respond to the class as a whole, (2) discriminate events which do or do not belong to a particular class of events, and (3) respond to events in terms of some common abstract property. Concepts are mental structures which have concrete references and free thought from the domination of the physical environment by generalizing to a total class of events. Examples of exercises measuring a child's arithmetic knowledge at this level are the following:

1. Join a set of five objects with a set of three objects and write a number sentence describing this action
2. Use objects to show how to find the answer to the number sentence: $3 + 4 = \square$

Thirdly, at the principle level of learning, a child combines two or more concepts to form relationships or principles. A prerequisite to the successful formation of principles is that the child must have knowledge of the concepts forming the principles. Furthermore, these principles are often reduced to verbal statements which the child remembers with little loss over periods of time. Examples of exercises measuring a child's arithmetic knowledge at this level are the following:

1. Write a number sentence to show how the numeral, 56, may be rewritten to form more ones.

2. Multiply the three factors, 4, 5, and 2, by grouping them in two different ways. Write number sentences illustrating the grouping procedures.

Lastly, at the problem solving level of learning, a child combines two or more principles in such a fashion that new relationships or higher-order principles become an integral part of his cognitive structure. Implicit within a child's functioning at this level is his ability to recall and apply relevant principles. Sometimes, verbal clues are given by the teacher to guide the child's thinking so that he is able to see relationships among principles. Furthermore, the derived relationships generalize to an entire class of problems. An example of exercises measuring a child's arithmetic knowledge at this level is the following:

> Use objects to show how to find the answer to the number sentence, 5/5 = /___/. Then, write an answer to the number sentence, 96/96 = /___/. Formulate a statement describing the solution of problems of this class.

Experimenter Designed Diagnostic Test

The hierarchies of mathematical skills together with the behavioral objectives defining each skill in the hierarchies served as a model for the experimenter while preparing the diagnostic test. For each skill in the hierarchies, three exercises were included in the test. These exercises required the child to: (1) manipulate physical objects showing how he arrived at an answer for a number sentence, (2) write a number sentence describing in numerical terms the action that an examiner performed with physical objects, and (3) use paper and pencil for computing the answer for a number sentence. Furthermore, exercises to measure a child's knowledge of the addition, subtraction, multiplication, and division combinations were written into the test.

Vocabulary was not a crucial variable to be tested with this instrument, and an attempt was made to use only the vocabulary which a child might normally be expected to know. Moreover, children were not required to read any instructions or written problems.

A teacher's guide was prepared to accompany the children's copy of the diagnostic test. In the teacher's guide were explicit directions stating: (1) the physical objects or conditions necessary for the performance of the exercise, (2) the method for manipulating the physical objects, and (3) the directions to be stated prior to the child performing an exercise.

Prior to the beginning of the experiment, the Experienced Teacher Fellowship Program Fellows and the fifth grade teachers in the twelve schools participating in the experiment selected several fifth grade children with learning difficulties in arithmetic. This "pilot" diagnostic test was administered individually to forty-eight selected fifth grade children. As the child performed the exercises on the test, the Fellows wrote suggestions on the teacher's guide for improving, deleting, or adding exercises to the diagnostic test. Particular attention was given to the child's response to the directions which were read to him.

After examining the suggestions made by the Fellows and the performance of the children, the experimenter met with the Fellows to revise the diagnostic test. Directions were rewritten and items which were successfully or unsuccessfully completed by nearly every child were deleted. Other items were revised by using smaller numbers or changing the position of the placeholder. Time limits were established for the items designed to measure a child's knowledge of the basic combinations for each operation. Further, it was recommended that larger type be used when the final diagnostic test was prepared.

Out of this session came the diagnostic test which was used in the experiment. It contains fewer items than the "pilot" test, but it still retains the essential characteristics of the "pilot" diagnostic test. These characteristics are the following.

1. Items to measure the child's knowledge of each skill in the hierarchy of arithmetic skills.

2. Items requiring the child to manipulate physical objects showing how he found the answer to the number sentence.

3. Items requiring the child to write a number sentence describing the action performed on physical objects by the examiner.

4. Items requiring the child to compute an answer to a problem using paper and pencil.

A copy of the revised diagnostic test and accompanying teacher's guide is found in Appendix C.

The exercises in the experimenter-designed arithmetic diagnostic test may be categorized under three broad areas: (1) concepts,

(2) place value, and (3) computation. Concepts category contains exercises requiring children to manipulate objects illustrating a number sentence and to write a number sentence describing the action performed on physical objects by the examiner. The test contains twenty eight exercises of this type: (1) five for addition, (2) seven for subtraction, (3) ten for multiplication, and (4) six for division.

Exercises in the place value category required children to manipulate objects illustrating regrouping procedures, to write number sentences describing regrouping procedures performed by the examiner on physical objects, to indicate the place value of a position occupied by a particular digit in a numeral, and to perform regrouping procedures on paper.

Computation category contained exercises requiring children to compute answers to problems using appropriate algorithms. These exercises involved the use of paper and pencil. Exercises were included which measured a child's computational skills for the four operations on whole numbers. The number of exercises for each operation was the following: (1) thirteen for addition, (2) thirteen for subtraction, (3) twenty for multiplication, and (4) twenty eight for division.

Knowledge of basic combinations for each operation was also measured. Not all of the basic combinations were included in the test, but rather a sampling of the combinations which often cause children difficulties. The number of combinations contained in the test was: (1) forty five for addition, (2) fifty for subtraction, (3) fifty for multiplication, and (4) fifty for division.

Experimenter-Designed Achievement Test

Essentially, there were three parts to the test: (1) concepts, (2) computation, and (3) applications. The concepts section of the test was designed to measure a child's knowledge of numerical concepts, principles, and their inter-relationships. Fifteen items were included in this section. On six items the test examiner manipulated objects and asked the child to write a number sentence describing the action performed by the examiner. Seven items required the child to write number sentences describing a given array. The remaining item in this section required the child to use a line to separate an array into two parts illustrating a given number sequence.

The computational section contained forty-two items requiring a child to apply principles, number facts, and algorithms to solve problems. Eleven items were designed to measure a child's knowledge of place value. Children were required to compute eleven addition problems, four subtraction problems, ten multiplication problems, and six division problems.

Four items were included in the applications section of the test: one item for each of the four mathematical operations. The vocabulary and reading level was kept to a low level in order to minimize the effects of a child's reading ability upon his arithmetic achievement.

A teacher's guide was developed to accompany the test. Directions to be verbally given to the child were stated in the guide, since the child was not required to read any directions. Furthermore, explicit directions were given to the examiner regarding the manipulation of the physical objects required by items in the concepts section. A copy of the experimenter-designed achievement test and its accompanying teacher's guide are found in Appendix C.

Attitude Scale

The attitude scale used in the study was designed to measure a child's attitudes toward: (1) arithmetic, (2) school, (3) reading, (4) spelling, and (5) science.

It is important to know the impact of the diagnostic and remedial programs upon the arithmetic achievement of fourth grade children. Equally important is the effect of the diagnostic and remedial programs upon the attitude development of the children. Therefore, the experimenter was interested in determining the effects of the two treatments upon children's attitudes toward arithmetic and school.

Furthermore, the experimenter was interested in determining the effects of the diagnostic and remedial programs upon attitudes toward other school subjects. For instance, much of the work in the Textbook Treatment required reading of an arithmetic textbook. Moreover, the experimenter has observed that children with learning difficulties in arithmetic often were not very successful in reading. These children often display poor attitudes toward both subjects. Thus, the effects of the two diagnostic and remedial treatments upon children's attitudes toward reading were investigated.

Likewise, at the elementary school level, the study of science often involves small group work, manipulation of physical objects, search for patterns, and applications of concepts, principles, and processes. Since the Laboratory Treatment had many of these same characteristics, the experimenter chose science.

The other school subject chosen by the experimenter for investigating the effects of the diagnostic and remedial programs upon attitude development was spelling. Often, the study of spelling involves repetition, drill, and rote memorization. Also, spelling textbooks usually provide a major portion of the instructional activities for children. These characteristics are descriptive of the Textbook Treatment.

Osgood's (1953) Semantic Differential served as a model for the development of the attitude scales. Eleven bi-polar items were chosen for each of the five attitude scales. In order to permit children to express strengths of feeling about each area under consideration, seven blanks were placed between each of the bi-polar items. The position of the positive and negative choices on each scale was alternated to stop a child from expressing a completely positive or negative attitude by checking all the items on one side without really reading the choices. Because they were not appropriate to the area being investigated, bi-polar items three and eight were included to break up a pattern of thought by acting as distractors.

Copies of the attitude scales used in this experiment are found in Appendix D.

Administration of the Evaluative Instruments

At the initiation of the experiment, three tests were administered by Experienced Teacher Fellowship Program Fellows to every child involved in the experiment: (1) Comprehensive Test of Basic Skills, Form Q1, (2) Stanford Diagnostic Arithmetic Test, and (3) experimenter-designed diagnostic arithmetic test. Furthermore, the tests were administered in the order listed. The administration dates are given in Table 3.12. Within each school, the standardized achievement and diagnostic tests were administered in a group situation. On the other hand, the experimenter-designed diagnostic test was administered to children on an individual basis or a small group situation; two or three children.

Upon completion of the instructional phase, two tests and the attitude scales were administered by the Fellows to every child: (1) Comprehensive Test of Basic Skills: Arithmetic, Form R1, (2) experimenter-designed arithmetic achievement test, and (3) attitude scales for arithmetic, school, reading, spelling, and science. The administration dates for these tests are given in Table 3.13. The attitude scales were administered prior to the achievement tests. Attitude scales, standardized achievement test, and two sections of the experimenter-designed achievement test, computational skills and problem solving, were administered in a group situation. However, the manipulation section of the experimenter-designed achievement test was administered on an individual basis or a small group situation; two or three children.

TABLE 3.12

PRE-TEST ADMINISTRATION SCHEDULE

Test	Days	Dates
Comprehensive Tests of Basic Skills: Arithmetic Form Q1	2	March 31 and April 1, 1970
Stanford Diagnostic Arithmetic Test	2	April 2 and April 3, 1970
Experimenter-Designed Diagnostic Test	4	April 6, 7, 8, and 9, 1970

TABLE 3.13

POST-TEST ADMINISTRATION SCHEDULE

Test	Days	Dates
Attitude Scales	1	May 28, 1970
Comprehensive Test of Basic Skills: Arithmetic, Form R1	2	June 1 and 2 1970
Experimenter-Designed Achievement Test	2	Juen 3 and 4, 1970

Reliability of the Test Instruments

Reliability estimates were computed using the Kuder-Richardson Formula 21 (Thorndike and Hagen, 1961). An essential assumption which is made when using the Kuder-Richardson reliability estimate is that the items in the test are homogeneous in the sense that every item measures the same general factors of ability or personality as do the others. It was due to this assumption that the experimenter computed reliability estimates for each portion of the achievement tests and diagnostic tests.

The reliability estimates for the experimenter-designed achievement test and its subtests are shown in Table 3.14. Variances and mean scores were included since the Kuder-Richardson Formula 21 required their use. The reliability coefficients for this test and its various subtests range from a coefficient of .51 on the manipulations subtest to a coefficient of .85 for the total test. All of these reliability coefficients were in excess of the minimum required to evaluate levels of group accomplishments as suggested by Kelly and quoted by Lindquist (1951). A minimum reliability coefficient of .50 was suggested as being necessary to evaluate group levels of accomplishments.

For the experimenter-designed diagnostic arithmetic test, reliability coefficients vary from a low of .70 on the concepts subtest to a high of .94 on the division subtest. All of the reliability coefficients were in excess of the .50 reliability coefficient required for evaluating group levels of accomplishments. Table 3.15 contains the mean scores, variances, and reliability coefficients for the total test as well as for the subtests.

Reliability coefficients were computed for the standardized achievement and diagnostic tests from the data obtained in the study. These tests were standardized and reliability coefficients computed prior to publication. Because this study involved only children with learning difficulties in arithmetic, the experimenter felt that computed reliability coefficients calculated from the experimental data would be of value when conclusions were formulated.

Tables 3.16, 3.17, and 3.18 show the computed reliability coefficients from data obtained in this study for the commercial diagnostic and achievement tests. Publishers' reliability coefficients are shown in Table 3.19.

The published reliablity coefficients for the diagnostic test were computed using the split-half (odd-even) method. To obtain the reliability coefficients for the two achievement tests, the Kuder-Richardson Formula 20 was used. By comparing the figures shown in the four tables, it can be seen that the experimental and published reliability coefficients for each test were very similar. In all

TABLE 3.14

RELIABILITY ESTIMATES, MEAN SCORES, AND VARIANCES ATTAINED ON THE EXPERIMENTER-DESIGNED ARITHMETIC ACHIEVEMENT TEST

Test	Treatment	Mean	Variance	Reliability
Manipulations	Textbook	10.26	6.81	.56
	Laboratory	11.01	5.61	.51
	Total	10.64	6.35	.54
Computation	Textbook	21.41	57.91	.83
	Laboratory	18.31	41.22	.76
	Total	19.84	51.84	.81
Applications	Textbook	1.57	1.82	.63
	Laboratory	1.72	1.71	.57
	Total	1.65	1.76	.60
Total Test	Textbook	33.24	98.01	.85
	Laboratory	31.04	73.27	.80
	Total	32.13	86.68	.83

TABLE 3.15

RELIABILITY ESTIMATES, MEAN SCORES, AND VARIANCES ATTAINED ON THE EXPERIMENTER-DESIGNED ARITHMETIC DIAGNOSTIC TEST

Test	Treatment	Mean	Variance	Reliability
Concepts	Textbook	12.65	21.62	.70
	Laboratory	13.59	21.53	.70
	Total	13.12	21.81	.70
Place Value	Textbook	4.62	10.24	.75
	Laboratory	4.96	13.47	.82
	Total	4.79	11.83	.79
Addition	Textbook	43.58	99.20	.90
	Laboratory	47.08	58.52	.86
	Total	45.32	82.08	.89
Subtraction	Textbook	29.34	128.37	.89
	Laboratory	27.42	162.56	.91
	Total	28.39	146.17	.90
Multiplication	Textbook	18.95	160.78	.92
	Laboratory	18.64	192.38	.94
	Total	18.80	176.62	.93
Division	Textbook	6.81	98.21	.94
	Laboratory	7.03	79.21	.93
	Total	6.92	88.74	.94
Total Test	Textbook	115.95	1153.96	.94
	Laboratory	118.73	1275.92	.94
	Total	117.33	1216.61	.94

TABLE 3.16

RELIABILITY ESTIMATES, MEAN SCORES AND VARIANCES ATTAINED ON THE STANFORD DIAGNOSTIC ARITHMETIC TEST

Test	Treatment	Mean	Variance	Reliability
Operations	Textbook	13.86	31.92	.82
	Laboratory	12.78	23.52	.75
	Total	13.31	27.98	.79
Place Value	Textbook	16.23	30.25	.77
	Laboratory	14.69	27.35	.75
	Total	15.45	29.38	.77
Addition	Textbook	14.99	11.97	.83
	Laboratory	14.54	10.11	.76
	Total	14.76	11.02	.80
Subtraction	Textbook	8.39	21.44	.83
	Laboratory	8.00	20.98	.83
	Total	8.20	21.25	.83
Multiplication	Textbook	5.04	20.88	.87
	Laboratory	3.79	14.98	.84
	Total	4.41	18.32	.86
Division	Textbook	2.25	5.85	.70
	Laboratory	1.82	3.80	.60
	Total	2.03	4.88	.66
Total Test	Textbook	60.76	348.20	.91
	Laboratory	55.63	274.56	.89
	Total	58.17	317.91	.90

TABLE 3.17

RELIABILITY ESTIMATES, MEAN SCORES AND VARIANCE ATTAINED ON COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM 01

Test	Treatment	Mean	Variance	Reliability
Computation	Textbook	36.71	143.04	.91
	Laboratory	35.41	132.48	.90
	Total	36.06	138.30	.91
Concepts	Textbook	16.81	40.70	.84
	Laboratory	15.53	37.21	.82
	Total	16.17	39.31	.83
Applications	Textbook	10.55	24.70	.84
	Laboratory	10.33	18.66	.77
	Total	10.44	21.72	.81
Total Test	Textbook	64.07	416.16	.94
	Laboratory	61.27	342.25	.92
	Total	62.67	381.03	.93

TABLE 3.18

RELIABILITY ESTIMATES, MEAN SCORES AND VARIANCES ATTAINED ON COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC FORM R1

Test	Treatment	Mean	Variance	Reliability
Computation	Textbook	43.71	121.22	.91
	Laboratory	42.42	101.00	.89
	Total	43.06	111.30	.90
Concepts	Textbook	18.50	32.26	.80
	Laboratory	17.51	30.47	.78
	Total	18.00	31.58	.79
Applications	Textbook	11.47	23.33	.83
	Laboratory	10.90	22.18	.81
	Total	11.18	22.85	.82
Total Test	Textbook	73.69	341.51	.93
	Laboratory	70.83	298.50	.92
	Total	72.24	321.84	.93

TABLE 3.19

PUBLISHERS' RELIABILITY COEFFICIENTS FOR ACHIEVEMENT AND DIAGNOSTIC TESTS

Test	Subtest	Reliability
Stanford Diagnostic Arithmetic Test	Operations	.89
	Place Value	.92
	Addition	.79
	Subtraction	.93
	Multiplication	.93
	Division	.77
	Total Test	.90
Comprehensive Test of Basic Skills, Form Q1	Computation	.88
	Concepts	.82
	Applications	.79
	Total Test	.92
Comprehensive Test of Basic Skills, Form R1	Computation	.94
	Concepts	.86
	Applications	.85
	Total Test	.96

instances, the experimental reliability coefficients for the three tests and their subtests were above the minimum required to evaluate group levels of accomplishment.

Indices of validity were not computed. Content validity for the experimenter-designed achievement and diagnostic tests was determined by carefully comparing the mathematics curricula of the three school districts with items in the two tests.

ANALYSIS OF RESULTS

Each student in the experiment was given a code number which indicated the school, treatment and student. This code number and that student's performance scores for each of the tests and attitude scales were key punched onto computer cards. A separate stack of cards was made for each test and the attitude scale. Table 3.21 lists the various stacks of cards and the information each card contained.

At the initiation of the study, children were administered a standardized arithmetic test and an experimenter-constructed diagnostic arithmetic test. A one-way analysis of variance was used to determine whether significant differences existed between mean scores achieved by treatment groups. Differences in mean scores were not significant at the .05 level of significance. Tables 4.01 - 4.06 show the analysis of variances of pre-tests results.

Since two sources of variation were studied, treatments and schools, two-way analysis of variance techniques were used to determine whether differences attributable to the sources of variation were significant. The measures analyzed were: (1) Comprehensive Test of Basic Skills: Arithmetic Battery and its subtests, (2) experimenter-constructed arithmetic achievement battery and its subtests and (3) attitude scales for arithmetic, school, reading, spelling and science. A fixed effect model with unequal cell frequencies was used in the analysis of the data. Since unequal and non-proportional cell frequencies resulted from reasons unrelated to the nature of the treatments, an unweighted means analysis was applied to data obtained from these tests.

Children in the study were selected from one or more fourth grade classrooms within a school. Then they were randomly assigned to treatments. Thus, the experimenter chose individuals as the experimental unit.

TABLE 3.20

STACKS OF COMPUTER CARDS AND INFORMATION ON EACH CARD

Stack 1 Comprehensive Test of Basic Skills: Arithmetic, Form Q1

Computational Score	Columns	1	-	4
Concepts Score		5	-	8
Applications Score		9	-	12
Total Score		13	-	16
Student Number		33	-	38

Stack 2 Stanford Diagnostic Arithmetic Test

Operations Score	Columns	1	-	4
Place Value Score		5	-	8
Addition score		9	-	12
Subtraction Score		13	-	16
Multiplication Score		17	-	20
Division Score		21	-	24
Total Score		25	-	28
Student Number		33	-	38

Stack 3 Experimenter Designed Diagnostic Test

Concepts Score	Columns	1	-	4
Place Value Score		5	-	8
Addition Score		9	-	12
Subtraction Score		13	-	16
Multiplication Score		17	-	20
Division Score		21	-	24
Total Score		25	-	28
Student Number		33	-	38

Stack 4 Comprehensive Test of Basic Skills: Arithmetic, Form R1

Computational Score	Columns	1	-	4
Concepts Score		5	-	8
Applications Score		9	-	12
Total Score		13	-	16
Student Number		33	-	38

Stack 5 Experimenter Designed Achievement Test

Manipulation Score	Columns	1 - 4
Computational Score		5 - 8
Applications Score		9 - 12
Total Score		13 - 16
Student Number		33 - 38

Stack 6 Attitude Scale

Arithmetic	Columns	1 - 4
School		5 - 8
Reading		9 - 12
Spelling		13 - 16
Science		17 - 20
Student Number		33 - 38

Stack 7 Computation of Reliability Coefficients

Number of Test Items	Columns	1 - 7
Mean Score		8 - 14
Standard Deviation		15 - 21
Test Number		29 - 32

LIMITATIONS OF THE STUDY

Since the schools involved in the study were not randomly chosen, the conclusions are limited to populations of schools whose characteristics are similar to the fourth grade children involved in this experiment.

Furthermore, the conclusions may not be applicable to children from other grade levels or from different backgrounds than the children involved in the experiment.

Moreover, the conclusions of this experiment may be applied only to children with learning difficulties in arithmetic and may not be applicable to children who are not consistently experiencing learning difficulties in arithmetic.

The results of this experiment are based upon the comparison of only two diagnostic and remedial instructional programs. Conclusions from this experiment may not be extended to other diagnostic and remedial programs.

In addition, the treatment period was of a relatively short duration, covering a period of seven weeks. No attempt has been made to extrapolate the results over a longer duration, such as an entire school year.

The attitude scales, one achievement test and one diagnostic test were designed by the experimenter for the purposes of this experiment. Furthermore, no standardization procedures were used.

CHAPTER IV

RESULTS OF THE INVESTIGATION

Introduction

Chapter four is divided into two main parts. The first section discusses the results as classified by pre-test evaluative instrument; the second section elaborates upon the results of administering post-test evaluative instruments. Sections one and two summarize the tests of significance and present supporting data in tabular form.

All of the evaluative instruments were administered by Experienced Teacher Fellowship Program Fellows. Except for the attitude scales, all tests were hand scored by the Experienced Teacher Fellowship Program Fellows, using scoring guides supplied by the test publisher or the experimenter. The evaluative instruments were collected by the experimenter who reviewed the scoring of each test. Furthermore, all attitude scales were hand scored by the experimenter. All of the results were recorded on master tabulating sheets for each treatment within a school. The master sheets contained the following information and scores:

1. School
2. Student's name and number
3. Comprehensive Test of Basic Skills: Arithmetic Test, Form Q1
    - A. Computation test
    - B. Concepts test
    - C. Applications test
    - D. Total test results
4. Stanford Diagnostic Arithmetic Test
    - A. Knowledge of operations
    - B. Knowledge of place value
    - C. Addition
    - D. Subtraction
    - E. Multiplication
    - F. Division
    - G. Total test results
5. Experimenter Designed Diagnostic Test
    - A. Concepts test
    - B. Place value test
    - C. Addition
    - D. Subtraction
    - E. Multiplication

  - F. Division
  - G. Total test results
6. Comprehensive Test of Basic Skills: Arithmetic Form R1
  - A. Computation test
  - B. Concepts test
  - C. Applications test
  - D. Total test
7. Experimenter-Designed Achievement Test
  - A. Concepts (Manipulative) test
  - B. Computation test
  - C. Applications test
  - D. Total test
8. Attitude Toward Arithmetic Scale
9. Attitude Toward School Scale
10. Attitude Toward Reading Scale
11. Attitude Toward Spelling Scale
12. Attitude Toward Science Scale

Data were punched onto cards by the experimenter. The analysis of data was accomplished at the University of Oregon Computer Center, using programs written by the experimenter. After receiving a print-out of the analyses, the experimenter used a calculator to check the analysis of data from two tests; thereby insuring that the program was functioning correctly.

Analysis of variance was the basic statistical technique used to test the variables under study and to test whether a significant difference in mathematical knowledge existed between treatment groups at the initiation of the experiment. For the analyses of data, students were used as the experimental unit. The statistical analysis of the data followed these steps:

A. Pre-test data
  1. Compute one-way analysis of variance (unequal frequencies per cell), as measured by Comprehensive Test of Basic Skills: Arithmetic, Form Q1, on:
     - a. Computation scores
     - b. Concepts scores
     - c. Application scores
     - d. Total test scores
  2. Compute one-way analysis of variance (unequal frequencies per cell), as measured by Stanford Diagnostic Arithmetic Test, on scores for the total battery.
  3. Compute one-way analysis of variance (unequal frequencies per cell), as measured by experimenter designed diagnostic arithmetic test, on scores for the total battery.

B. Post-test Data
  1. Compute two-way analysis of variance (fixed effects model with unequal frequencies per cell), as measured

by Comprehensive Test of Basic Skills: Arithmetic, Form R1, on:
   a. Computation scores
   b. Concepts scores
   c. Application scores
   d. Total test scores
2. Compute two-way analysis of variance (fixed effects model with unequal frequencies per cell), as measured by experimenter designed arithmetic test on:
   a. Concepts (manipulative scores),
   b. Computation scores
   c. Applications scores
   d. Total test scores
3. Compute two-way analysis of variance (fixed effects model with unequal frequencies per cell) as measured by attitude scale on:
   a. Attitude toward arithmetic scores
   b. Attitude toward school scores
   c. Attitude toward reading scores
   d. Attitude toward spelling scores
   e. Attitude toward science scores

FINDINGS SUMMARIZED ACCORDING TO STUDY INSTRUMENTS

Pre-Test Measures

Using analysis of variance techniques with unequal frequencies per cell, an F was computed on scores obtained from the administration of Comprehensive Test of Basic Skills: Arithmetic, Form Q1, Stanford Diagnostic Arithmetic Test, and experimenter-designed diagnostic arithmetic test. The purpose of this was to test the hypothesis that the treatment groups were not significantly different in their knowledge of arithmetic processes at the initiation of the study.

Tables 4.01, 4.02, and 4.03 show the analysis of variance for the subtests and total battery of the Comprehensive Test of Basic Skills: Arithmetic, Form Q1. No significant difference was found between treatment groups in knowledge of arithmetic processes as measured by the subtests and total arithmetic battery. Tables 4.05 and 4.06 show the analysis of variance for the total battery of the Stanford Diagnostic Arithmetic Test and the experimenter-designed diagnostic arithmetic test, respectively. For each diagnostic test, no significant difference was found between treatment groups in knowledge of arithmetic processes.

TABLE 4.01

ANALYSIS OF VARIANCE FOR COMPUTATIONAL SKILLS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM 01

SV	DF	SS	MS	F
Treatments	1	61.81	61.81	.442
Error	144	20119.87	139.72	
Total	145	20181.68		

$F_{.05(1,144)} = 3.91$

TABLE 4.02

ANALYSIS OF VARIANCE FOR KNOWLEDGE OF CONCEPTS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM 01

SV	DF	SS	MS	F
Treatments	1	59.21	59.21	1.50
Error	144	5679.09	39.44	
Total	145	5738.30		

$F_{.05(1,144} = 3.91$

TABLE 4.04

ANALYSIS OF VARIANCE FOR COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM Q1

SV	DF	SS	MS	F
Treatments	1	284.88	284.88	.74
Error	144	55364.87	384.88	
Total	145	55649.75		

$F_{.05\ (1,144)} = 3.91$

TABLE 4.05

ANALYSIS OF VARIANCE FOR THE STANFORD DIAGNOSTIC ARITHMETIC TEST

SV	DF	SS	MS	F
Treatments	1	942.75	942.75	2.99
Error	144	44509.31	315.67	
Total	145	45452.06		

$F_{.05(1,144)} = 3.91$

TABLE 4.06

ANALYSIS OF VARIANCE FOR THE EXPERIMENTER-DESIGNED DIAGNOSTIC ARITHMETIC TEST

SV	DF	SS	MS	F
Treatments	1	284.00	284.00	0.231
Error	145	178519.00	1231.16	
Total	146	178803.00		

$F_{.05(1,144)} = 3.91$

POST-TEST DATA

Results of the analyses of the post achievement tests and post attitude scales are reported in this section. Each of the achievement tests were analyzed in sections as well as the total battery. The three sections of each achievement test are computation, concepts, and applications.

Two-way analysis of variance technique using a fixed model with unequal cell frequencies was chosen to analyze post test data for several reasons. First, two sources of variation were studied; treatments and schools. Secondly, treatment levels were determined in a systematic, non-random fashion. Furthermore, all treatment levels about which inferences were to be drawn were included in the study. Thirdly, although the original design called for equal frequencies in each treatment cell, unequal frequencies in treatment cells resulted at the termination of the study. These unequal cell frequencies occurred because: (1) children were absent on days of final testing or (2) children moved from the school's residential area.

Moreover, since these unequal cell frequencies were not proportional and were not related to the nature of the treatments, an unweighted means analysis procedure was used to analyze the post-test data.

Sources of variation are treatment and school effects. Within each of twelve schools, twelve fourth grade children with learning difficulties in arithmetic were selected and randomly assigned to treatments by drawing their names from a box. To control for school environment the two distinct treatments were applied in each school.

Furthermore, teacher effect was accounted for by randomization, and teachers were closely supervised during the experiment to ascertain that treatment descriptions were followed. Prior to and during the experiment, teachers of both groups received the same training and instructions regarding the experiment.

Since the two sources of variation, treatments and schools, are fixed, the mean square within error term is the proper denominator for each F-ratio in the analysis of variance for post-test data.

ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC

Computation Subtest

Table 4.07 shows the analysis of variance for the computational skills subtest of the Comprehensive Test of Basic Skills: Arithmetic, Form R1. Differences among schools are significant at the .01 level.

Table 4.08 shows mean scores by school and treatment. In eight schools, mean scores reveal differences favoring the Textbook Treatment over the Laboratory Treatment. In most cases, the differences are small. However, in Schools 3 and 10 differences in mean scores are 8 1/2 and 9 points, respectively.

Concepts Subtest

Table 4.09 shows the analysis of variance for the knowledge of concepts subtest of the Comprehensive Test of Basic Skills: Arithmetic, Form R1. Differences which are significant are limited to only one item, schools. This difference is significant at the .05 level of significance.

Examination of the mean scores in Table 4.10 reveals that the mean scores within eight schools favor the Textbook Treatment. The largest differences between mean scores occur in Schools 8 and 9. In both schools the difference between mean scores is approximately 6.4 points. However, in School 8 this difference favors the Textbook Treatment while the difference in mean scores for School 9 favors the Laboratory Treatment.

Applications Subtest

Table 4.11 shows the analysis of variance for the applications subtest of the Comprehensive Test of Basic Skills: Arithmetic, Form R1. Differences which are significant are limited to the one item, schools. This difference is significant at the .05 level of significance.

Examination of the mean scores shown in Table 4.12 reveals that the mean scores favor the Textbook Treatment in seven schools. Most of the differences between mean scores are small, approximately 2 1/2 points, with the largest difference of approximately 4.3 points occurring in Schools 6 and 7. In School 6 the mean score for the Textbook Treatment is the larger while in School 7 the Laboratory Treatment has the larger mean score.

Comprehensive Test of Basic Skills: Arithmetic Battery

Examination of Table 4.13 reveals that the trend which was evident in Tables 4.11, 4.09 and 4.07 is continued in the analysis of variance for the Comprehensive Test of Basic Skills: Arithmetic, Form R1. The only significant difference is the one item, schools, which is significant at the .01 level of significance.

Examination of mean scores shown in Table 4.14 reveals that the mean scores favor the Textbook Treatment in nine schools. In most of the schools, the differences between mean scores are relatively small, approximately five points. However, the differences between mean scores for Schools 3, 7, 8, 9, and 10 are approximately thirteen, thirteen, twelve, fifteen and eleven points, respectively. Of these five schools, the mean scores in Schools 7 and 9 favor the Laboratory Treatment while the Textbook Treatment had the larger mean scores in Schools 3, 8, and 10.

TABLE 4.07

ANALYSIS OF VARIANCE FOR COMPUTATIONAL SKILLS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

SV	DF	SS	MS	F
Treatments	1	71.265	71.265	.715
Schools	11	3265.872	296.898	2.978*
Interaction	11	773.550	70.323	.705
Within Error	118	11763.100	99.687	

$F_{.05\ (1,118)} = 3.92$ $\quad$ $*F_{.01\ (11,118)} = 2.40$

TABLE 4.08

MEAN SCORES BY SCHOOL AND TREATMENT FOR COMPUTATIONAL SKILLS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

School	Laboratory Treatment	Textbook Treatment
1	42.50	44.00
2	41.50	39.83
3	32.17	40.67
4	44.60	45.00
5	36.00	41.83
6	40.33	36.00
7	46.40	40.67
8	47.80	51.00
9	39.83	34.67
10	39.83	48.83
11	50.83	53.80
12	47.33	50.20
Total	42.42	43.71

TABLE 4.09

ANALYSIS OF VARIANCE FOR KNOWLEDGE OF CONCEPTS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

SV	DF	SS	MS	F
Treatments	1	34.857	34.857	1.222
Schools	11	651.172	59.198	2.075*
Interaction	11	401.701	36.518	1.280
Within Error	118	3366.766	28.532	

$F_{.05(1,118)} = 3.92$ *$F_{.05(11,118)} = 1.87$

TABLE 4.10

MEAN SCORES BY SCHOOL AND TREATMENT FOR KNOWLEDGE OF CONCEPTS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

School	Laboratory Treatment	Textbook Treatment
1	15.25	16.00
2	13.50	17.50
3	11.83	15.00
4	18.60	21.30
5	19.33	18.00
6	17.50	22.00
7	19.00	16.33
8	17.40	23.80
9	22.17	15.83
10	15.50	16.50
11	17.50	19.60
12	21.33	19.20
Total	17.51	18.50

TABLE 4.11

ANALYSIS OF VARIANCE FOR APPLICATIONS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

SV	DF	SS	MS	F
Treatments	1	6.057	6.057	.284
Schools	11	465.234	42.294	1.984*
Interaction	11	221.852	20.168	.946
Within Error	118	2516.055	21.323	

$F_{.05(1,118)} = 3.92$ $*F_{.05(11,118)} = 1.87$

TABLE 4.12

MEAN SCORES BY SCHOOL AND TREATMENT FOR APPLICATIONS SUBTEST OF COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

School	Laboratory Treatment	Textbook Treatment
1	10.75	8.50
2	8.67	11.17
3	6.00	8.00
4	11.60	14.30
5	11.50	11.50
6	8.17	12.40
7	14.20	9.83
8	11.60	14.00
9	12.67	9.00
10	9.17	10.33
11	13.33	13.20
12	13.33	13.80
Total	10.90	11.47

TABLE 4.13

ANALYSIS OF VARIANCE FOR COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

SV	DF	SS	MS	F
Treatments	1	282.048	282.048	.969
Schools	11	8366.468	760.588	2.613*
Interaction	11	2468.980	224.453	.771
Within Cell	118	34352.440	291.122	

$F_{.05(1,118)} = 3.92$ $*F_{.01(11,118)} = 2.40$

TABLE 4.14

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE COMPREHENSIVE TEST OF BASIC SKILLS: ARITHMETIC, FORM R1

School	Laboratory Treatment	Textbook Treatment
1	68.50	68.50
2	63.67	68.50
3	50.00	63.67
4	74.80	80.60
5	66.83	71.33
6	66.00	70.40
7	79.60	66.83
8	76.80	88.80
9	74.67	59.50
10	64.50	75.67
11	81.67	86.60
12	82.00	83.20
Total	70.83	73.69

ANALYSIS OF EXPERIMENTER-DESIGNED ACHIEVEMENT TEST

Manipulation Subtest

Table 4.15 shows the analysis of variance for the manipulation (concept) subtest of the experimenter-designed arithmetic achievement test. Treatment effects were significant at .05 level of significance. Both school and interaction effects were significant at the .01 level of significance.

Table 4.16 shows the mean scores by school and treatment. In eight schools the Laboratory Treatment obtained the same or higher mean scores than the Textbook Treatment. The differences in mean scores are quite small, except for Schools 3 and 6 where the differences between mean scores is four points. In both schools, the Laboratory Treatment has the larger mean score.

Computational Subtest

Table 4.17 shows the analysis of variance for the computational skills subtest of the experimenter-designed arithmetic achievement test. Sources of variation which achieve significance are treatments and schools with F-ratios which are significant at the .01 level.

Examination of mean scores shown in Table 4.18 reveals that the mean scores consistently favor the Textbook Treatment in each school. In most schools the differences in mean scores are small. However, the difference between mean scores within Schools 8 and 10 are approximately eight and ten points, respectively.

Applications Subtest

Table 4.19 shows the analysis of variance for the applications subtest of the experimenter designed arithmetic achievement test. Sources of variation which are significant at the .01 level of significance are schools and interaction.

Examination of mean scores shown in Table 4.20 reveals that the mean scores in six schools favor the Laboratory Treatment. Differences between mean scores are quite small except for Schools 2 and 10 whose differences between mean scores are approximately 2.8 and 2 points, respectively. The mean scores favor the Laboratory Treatment in School 2 while the Textbook Treatment is favored in School 10.

Experimenter Designed Achievement Battery

Analysis of variance for the total battery of the experimenter-designed arithmetic achievement test is shown in Table 4.21. The source of variation that gave significant results is schools. School effect is significant at the .01 level of significance.

Examination of the mean scores shown in Table 4.22 reveals differences that favor the Textbook Treatment in seven schools. Most of the mean scores within schools are similar; however, Schools 8 and 10 have mean score differences of approximately 6 and 13 points, respectively.

TABLE 4.15

ANALYSIS OF VARIANCE FOR THE MANIPULATION SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

SV	DF	SS	MS	F
Treatments	1	26.122	26.122	6.364*
Schools	11	270.469	24.588	5.990**
Interaction	11	114.263	10.388	2.530**
Within Error	118	484.395	4.105	

$F_{.05(1,118)} = 3.92$ $**F_{.01(11,118)} = 2.40$

TABLE 4.16

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE MANIPULATION SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

School	Laboratory Treatment	Textbook Treatment
1	11.25	11.25
2	13.83	11.83
3	13.00	9.00
4	10.30	12.44
5	13.17	11.33
6	11.00	7.20
7	7.00	7.33
8	11.80	10.00
9	10.83	9.83
10	9.33	10.67
11	10.33	11.00
12	10.33	9.80
Total	11.01	10.26

TABLE 4.17

ANALYSIS OF VARIANCE FOR THE COMPUTATION SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

SV	DF	SS	MS	F
Treatments	1	347.652	347.652	8.008*
Schools	11	1646.940	149.722	3.449**
Interaction	111	312.255	28.387	.6539
Within Error	118	5122.582	43.412	

$^*F_{.01(1,118)} = 6.85$ $^{**}F_{.01(11,118)} = 2.40$

TABLE 4.18

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE COMPUTATION SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

School	Laboratory Treatment	Textbook Treatment
1	15.00	17.75
2	25.67	25.67
3	14.17	19.33
4	19.10	21.44
5	16.00	18.00
6	17.17	20.00
7	17.40	18.00
8	23.20	31.00
9	15.33	16.17
10	15.17	25.67
11	20.67	23.33
12	19.83	20.60
Total	18.31	21.41

TABLE 4.19

ANALYSIS OF VARIANCE FOR THE APPLICATION SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

SV	DF	SS	MS	F
Treatments	1	1.381	1.381	1.094
Schools	11	56.879	5.171	4.096*
Interaction	11	48.002	4.364	3.457*
Within Error	118	148.956	1.262	

$F_{.05(1,118)} = 3.92$ $^*F_{.01(11,118)} = 2.40$

TABLE 4.20

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE APPLICATIONS SUBTEST OF THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

School	Laboratory Treatment	Textbook Treatment
1	1.25	0.25
2	3.83	1.00
3	0.83	2.00
4	1.40	1.56
5	1.00	0.83
6	1.83	1.60
7	2.80	1.17
8	2.80	3.00
9	1.17	0.50
10	0.67	2.50
11	2.17	2.67
12	1.33	1.60
Total	1.72	1.57

TABLE 4.21

ANALYSIS OF VARIANCE FOR THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

SV	DF	SS	MS	F
Treatments	1	152.744	152.744	2.093
Schools	11	2885.853	262.350	3.595*
Interaction	11	691.142	62.831	0.861
Within Error	118	8610.214	72.968	

$F_{.05(1,118)} = 3.92$ *$F_{.01(11,118)} = 2.40$

TABLE 4.22

MEAN SCORES BY SCHOOLS AND TREATMENT FOR THE EXPERIMENTER DESIGNED ARITHMETIC ACHIEVEMENT TEST

School	Laboratory Treatment	Textbook Treatment
1	27.50	29.25
2	43.33	38.50
3	28.00	30.33
4	30.80	35.44
5	30.17	30.17
6	30.00	28.80
7	27.20	26.50
8	37.80	44.00
9	27.33	26.50
10	25.17	38.83
11	33.17	37.00
12	31.50	32.00
Total	31.04	33.24

ANALYSIS OF ATTITUDE SCALES

Attitude Toward Arithmetic Scale

Table 4.23 shows the analysis of variance for the attitude toward arithmetic scale. Sources of variation which are significant at the .01 and .05 levels of significance are, respectively, schools and interaction.

Examination of mean scores shown in Table 4.24 reveals that the mean scores within schools vary considerably. The smallest difference between mean scores within schools is 1.37 in School 7 with the largest difference of 20.67 in School 9. Furthermore, mean scores favor the Laboratory Treatment in seven schools.

Attitude Toward School Scale

Table 4.25 shows the analysis of variance for the attitude toward school scale. No significant differences were found.

Examination of the mean scores shown in Table 4.26 reveals that mean scores favor the Laboratory Treatment in seven schools. In contrast to the differences between mean scores in the other schools, the difference between mean scores for School 9 is quite large, 23.33 points.

Attitude Toward Reading Scale

Analysis of variance for the attitude toward reading scale is shown in Table 4.27. Sources of variation which are significant at the .01 and .05 level of significance are treatments and interactions, respectively.

Examination of the mean scores shown in Table 4.28 reveals that mean scores favor the Laboratory Treatment in eight schools. In Schools 6, 8, 9, and 12 the differences between mean scores are approximately seventeen, twelve, eighteen and thirteen points, respectively. These differences are rather large in comparison to the differences between mean scores in the other schools.

Attitude Toward Spelling Scale

Table 4.29 shows the analysis of variance for the attitude toward spelling scale. No significant differences were found.

Examination of mean scores shown in Table 4.30 reveals that mean scores favor the Textbook Treatment in seven schools. The difference between mean scores in School 9 is quite large in comparison to the difference between mean scores in the other schools. This difference for School 9 is 17.50 points.

Attitude Toward Science Scale

Analysis of variance for the attitude toward science scale is shown in Table 4.31. The interaction source of variation is significant at the .01 level of significance.

Examination of the mean scores shown in Table 4.32 reveals that the mean scores favor the Textbook Treatment in seven schools. The largest differences in mean scores occurs in Schools 9 and 12 with differences of 22.50 and 15.70 points, respectively. In School 9 the mean scores favor the Laboratory Treatment; on the other hand, the mean scores in School 12 favor the Textbook Treatment.

TABLE 4.23

ANALYSIS OF VARIANCE FOR THE ATTITUDE TOWARD ARITHMETIC TEST

SV	DF	SS	MS	F
Treatments	1	219.686	219.686	1.372
Schools	11	4375.281	397.753	2.484*
Interaction	11	4138.945	376.268	2.350**
Within Error	118	18897.070	160.145	

$F_{.05(1,118)} = 3.92$ **$F_{.05(11,118)} = 1.87$

*$F_{.01(11,118)} = 2.40$

TABLE 4.24

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE ATTITUDE TOWARD ARITHMETIC TEST

School	Laboratory Treatment	Textbook Treatment
1	51.25	68.75
2	61.17	53.33
3	59.67	50.83
4	52.80	57.40
5	59.60	58.17
6	62.67	47.20
7	51.80	53.17
8	61.00	52.00
9	54.00	33.33
10	65.50	56.83
11	67.00	69.50
12	44.17	59.60
Total	57.47	54.85

TABLE 4.25

ANALYSIS OF VARIANCE FOR THE ATTITUDE TOWARD SCHOOL TEST

SV	DF	SS	MS	F
Treatments	1	190.630	190.630	1.129
Schools	11	2222.720	202.006	1.197
Interaction	11	2993.037	272.094	1.612
Within Error	118	19921.310	168.825	

$F_{.05(1,118)} = 3.92$ $f_{.05(11,118)} = 1.87$

TABLE 4.26

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE ATTITUDE TOWARD SCHOOL TEST

School	Laboratory Treatment	Textbook Treatment
1	61.00	70.25
2	61.50	59.17
3	65.17	59.50
4	58.70	55.70
5	68.60	60.83
6	62.67	53.60
7	54.60	60.33
8	67.00	55.60
9	62.50	39.17
10	58.83	66.17
11	61.50	64.50
12	51.33	60.20
Total	60.89	58.35

TABLE 4.27

ANALYSIS OF VARIANCE FOR THE ATTITUDE TOWARD READING TEST

SV	DF	SS	MS	F
Treatments	1	463.820	463.820	4.016*
Schools	11	1902.051	172.914	1.497
Interaction	11	2895.241	263.204	2.279**
Within Error	118	13626.870	115.482	

$^*F_{.05(1,118)} = 3.92$ $^{**}F_{.05(11,118)} = 1.87$

TABLE 4,28

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE ATTITUDE TOWARD READING TEST

School	Laboratory Treatment	Textbook Treatment
1	63.00	71.25
2	64.17	59.67
3	65.67	62.00
4	66.00	63.50
5	67.40	59.33
6	67.67	50.40
7	70.00	62.67
8	69.60	57.20
9	63.50	45.83
10	66.00	70.50
11	64.50	68.17
12	53.33	66.00
Total	65.01	61.37

TABLE 4.29

ANALYSIS OF VARIANCE FOR THE ATTITUDE TOWARD SPELLING TEST

SV	DF	SS	MS	F
Treatments	1	48.898	48.898	0.249
Schools	11	1739.412	158.128	0.804
Interaction	11	2810.911	255.537	1.299
Within Error	118	23218.610	196.768	

$F_{.05(1,118)} = 3.92$ $F_{.05(11,118)} = 1.87$

TABLE 4.30

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE ATTITUDE TOWARD SPELLING TEST

School	Laboratory Treatment	Textbook Treatment
1	54.00	67.25
2	60.33	48.33
3	68.17	59.17
4	57.80	58.30
5	65.20	56.50
6	64.33	57.80
7	60.20	61.00
8	59.00	65.20
9	65.17	47.67
10	61.67	64.50
11	62.67	68.00
12	48.00	58.40
Total	60.54	59.01

TABLE 4.31

ANALYSIS OF VARIANCE FOR THE ATTITUDE TOWARD SCIENCE TEST

SV	DF	SS	MS	F
Treatments	1	0.01134	0.01134	.00007
Schools	11	2227.681	202.517	1.197
Interaction	11	4685.324	425.939	2.517*
Within Error	118	19972.000	169.254	

$F_{.05(1,118)} = 3.92$

'$F_{.05(11,118)} = 1.87$

*$F_{.01(11,118)} = 2.40$

TABLE 4.32

MEAN SCORES BY SCHOOL AND TREATMENT FOR THE ATTITUDE TOWARD SCIENCE TEST

School	Laboratory Treatment	Textbook Treatment
1	48.75	65.00
2	69.50	62.33
3	58.33	60.17
4	69.90	61.40
5	62.60	66.00
6	71.33	60.80
7	55.00	64.67
8	69.60	57.40
9	68.00	45.50
10	68.50	70.50
11	57.17	69.00
12	49.50	65.20
Total	63.16	62.25

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Introduction

This study had its origin in the need for experimental evidence to determine the effects of a highly structured diagnostic and remedial arithmetic program upon elementary children. Using fourth grade children with learning difficulties in arithmetic, the study was conducted from March through June of the 1969-70 school year in two schools in the Bethel School District, one school in the Springfield School District and in nine schools in the Eugene School District.

SUMMARY

The purpose of this study was to compare the effects of two distinct diagnostic and remedial arithmetic programs upon the arithmetic achievement and attitude development of fourth grade children. Arithmetic achievement was determined through both a commerically standardized test and an experimenter-designed test. These achievement tests measured children's computational skill, knowledge of arithmetic concepts, and problem solving abilities (applications). Children's attitudes were measured by tests of attitude toward arithmetic, school, reading, spelling and science.

Two experimental treatments were administered to the children participating in the study. The Laboratory Treatment consisted of a remedial arithmetic program based upon the child's performance on an experimenter-designed diagnostic arithmetic test. First, hierarchies of arithmetical skills were constructed and each skill behaviorally defined. Then a diagnostic test was constructed which contained exercises designed to measure a child's knowledge of each skill in the hierarchies. These exercises required the child to manipulate objects illustrating a mathematical principle, to write arithmetical sentences describing the manipulation of objects by the examiner and to compute answers to exercises using pencil and paper. The remedial program began at the lowest skill in the hierarchy which the child could not perform as measured by the experimenter-constructed diagnostic test. Successive skills were taught as soon as lower level skills were mastered by the child. Furthermore, teachers designed instructional activities for each skill which the child had not acquired in the past. Instructional activities made extensive use of models, visual aids, games and puzzles. Moreover, children as well as the teacher manipulated the models illustrating arithmetic principles.

In contrast to this, the Textbook Treatment consisted of a remedial arithmetic program based upon the child's performance on the Stanford Diagnostic Arithmetic Test. A grade equivalent score was found in the publisher's manual that corresponded to the raw score obtained from the child's performance on the test. An arithmetic textbook whose grade level designation corresponded to the child's grade equivalent score was chosen by the teacher for the child to use in the study. After the teacher chose a place in the textbook which approximated the decimal part of the grade equivalent score, the child started working the exercises in the textbook. For the duration of the study, the child was permitted to perform only the instructional activities suggested in the textbook and its accompanying teacher's manual.

Two teachers participating in the Experienced Teacher Fellowship Program were randomly assigned to each of the twelve schools. Within each school, the two teachers randomly assigned themselves to the two treatments by a flip of a coin.

Schools involved in the study were the ones selected by the three districts' administrators and Experienced Teacher Fellowship Program directors to participate in the Experienced Teacher Fellowship Program. Schools were selected on the basis of: (1) willingness to participate in the Experienced Teacher Fellowship Program, (2) available space and facilities for the Experienced Teacher Fellowship Program Fellows to instruct children and (3) number of children with learning difficulties in arithmetic, reading and language arts.

Within each school, children with learning difficulties in arithmetic were identified to participate in the study by the regular fourth grade teachers and the two Experienced Teacher Fellowship Program Fellows. Basis for identification was the child's performance on: (1) daily arithmetic assignments, (2) teacher constructed arithmetic tests and (3) commercial achievement tests administered by the district. Children were randomly assigned to treatments by drawing their names out of a box.

Several measures of a child's achievement were made at the beginning of the study. These measures were: (1) Comprehensive Test of Basic Skills: Arithmetic Test, Form 01, (2) Stanford Diagnostic Arithmetic Test and (3) experimenter designed diagnostic arithmetic test. At the conclusion of the experiment, several measures of a child's achievement and attitude were made. These measures were: (1) Comprehensive Test of Basic Skills: Arithmetic Test, Form R1, (2) experimenter designed arithmetic achievement test and (3) attitude scales for arithmetic, school, reading, spelling and science.

All test instruments were administered and hand scored by the Experienced Teacher Fellowship Program Fellows with the exception of the attitude scales. Experienced Teacher Fellowship Program Fellows administered the attitude scales, but they were hand scored by the experimenter. The data collected was punched on computer cards, and the statistical analyses were computed by computer at the Computer Center on the University of Oregon campus.

Even though children were randomly assigned to treatments, the experimenter chose to make a one-way analysis of variance on the three sets of pre-experiment test data. Differences among means were not significant at the .05 level.

To analyze the data from tests administered at the conclusion of the study, the experimenter used a two-way analysis of variance for a fixed effects model with unequal cell frequencies. To accomplish this, an unweighted means procedure was used. Pupils were chosen as the experimental unit.

FINDINGS

Listed below are the thirteen hypotheses of the study and the conclusions of the experimenter based on statistical data:

Hypothesis No. 1 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. However, examination of the means revealed a slight advantage for those students receiving the Textbook Treatment over those receiving the Laboratory Treatment. The mean scores for the Laboratory and Textbook Treatments are 42.42 and 43.71, respectively.

Hypothesis No. 2 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the computational skills test of the experimenter-designed arithmetic achievement test.

The hypothesis is rejected. Significant differences attributable to treatments were found in the analysis of variance. The mean scores within schools consistently favored those students receiving the Textbook Treatment over those receiving the Laboratory Treatment. The mean scores for the Laboratory Treatment and the Textbook Treatment are 18.31 and 21.41, respectively.

Hypothesis No. 3 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on concepts test of the Comprehensive Test of Basic Skills: Arithmetic, Form R1.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. Examination of mean scores reveals a slight treatment effect favoring Textbook Treatment over the other treatment. The mean score for the Textbook Treatment is 18.50 while the mean score for the Laboratory Treatment is 17.51.

Hypothesis No. 4 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on manipulation (concepts) test of the experimenter-designed arithmetic achievement test.

The hypothesis is rejected. Significant differences attributable to treatments were found in the analysis of variance. Examination of mean scores within schools reveals a slight treatment effect favoring the Laboratory Treatment over the other treatment. The mean score for the Laboratory Treatment is 11.01 while the mean score for the Textbook Treatment is 10.26. However, since the difference between mean scores by treatment is less than one standard deviation, the experimenter concludes that it is not educationally significant.

Hypothesis No. 5 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on applications test of the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. The trend among the means scores showed a slight advantage for those students receiving Textbook Treatment over those receiving the Laboratory Treatment. Furthermore, the mean scores for the Laboratory Treatment and Textbook Treatment were 10.90 and 11.47, respectively.

Hypothesis No. 6 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on applications test of the experimenter-designed arithmetic achievement test.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. The trend among the means within schools showed a slight advantage for those students receiving the Laboratory Treatment over those receiving the Textbook Treatment. The mean score for the Laboratory Treatment was 1.72 while the Textbook Treatment had a mean score of 1.57.

Hypothesis No. 7 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the Comprehensive Test of Basic Skills: Arithmetic Battery, Form R1.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. Examination of mean scores within schools reveals a slight treatment effect favoring the Textbook Treatment over the other treatment. The mean score for the Textbook Treatment was 73.69 while the Laboratory Treatment had a mean score of 70.83.

Hypothesis No. 8 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean achievement on the experimenter-designed arithmetic achievement test battery.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. Examination of mean scores within schools reveals a slight treatment effect favoring Textbook Treatment over the Laboratory Treatment. Furthermore, the mean scores for the Textbook Treatment and the Laboratory Treatment are 33.24 and 31.04, respectively.

Hypothesis No. 9 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude toward arithmetic.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. The trend among the means within schools showed a slight advantage for those students receiving the Laboratory Treatment over those receiving the Textbook Treatment. Mean scores for the Laboratory Treatment and the Textbook Treatment were 57.47 and 54.85, respectively.

Hypothesis No. 10 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude toward school.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. The trend among the means within schools showed a slight advantage for those students receiving the Laboratory Treatment over those receiving the Textbook Treatment. The mean score for the Laboratory Treatment was 60.89 while the mean score for the Textbook Treatment was 58.35.

Hypothesis No. 11 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude toward reading.

The hypothesis is rejected. Significant differences attributable to treatments were found in the analysis of variance. The trend among the means within schools showed an advantage for those students receiving the Laboratory Treatment over those receiving the Textbook Treatment. Furthermore, the mean score for the Laboratory Treatment was 65.01 while the mean score for the Textbook Treatment was 61.37.

Hypothesis No. 12 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude toward spelling.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. The trend among the means showed a slight advantage for those students receiving the Laboratory Treatment over the other treatment. Mean scores for the Laboratory Treatment and the Textbook Treatment were 60.54 and 59.01, respectively.

Hypothesis No. 13 - No significant differences exist among fourth grade children receiving the Laboratory and Textbook Treatments in mean performance on an attitude scale designed to measure attitude toward science.

The hypothesis is not rejected. No significant differences attributable to treatments were found in the analysis of variance. Examination of mean scores reveals a treatment effect favoring the Laboratory Treatment over the Textbook Treatment. Furthermore, the mean score for the Laboratory Treatment was 63.16 while the Textbook Treatment had a mean score of 62.25.

LIMITATIONS OF THE STUDY

The following limitations should be considered when drawing conclusions based on the results of this experiment:

1. All conclusions which were derived in this study are limited to those populations which have the same characteristics as the population of this study.
2. All conclusions which are formulated in this study are applicable to fourth grade children who have mathematics

learning difficulties. No attempt should be made to apply the results of this study to other grade levels, to mentally retarded children, or to children who are at or above grade level in their mathematics achievement as measured by an appropriate achievement test.

3. All conclusions were based on the two specific treatments used in this study. Any attempt to apply these conclusions to extensions of the two treatments should be made with caution.

4. Some conclusions were formulated from the results of evaluative instruments designed specifically for this study.

5. All conclusions were made under the assumption that the teacher variable was equalized through the randomization process employed to assign teachers to schools and to treatments.

6. The instructional phase of the study lasted for seven weeks and consisted of approximately twenty-nine instructional periods. Any attempts to apply these conclusions to programs with different amounts of instructional time should be made with caution.

CONCLUSIONS

From the data obtained in this experiment, several conclusions were formulated for intermediate grade teachers, elementary school principals and other persons interested in diagnostic and remedial arithmetic programs for intermediate grade children. The following conclusions are applicable to the fourth grade population from which the children participating in this experiment were selected:

1. No significant differences by treatment exist among students' scores on the computational skills test of the Comprehensive Test of Basic Skills: Arithmetic Battery. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon computational skills of fourth grade children, as measured by the Comprehensive Test of Basic Skills: Arithmetic Battery.

2. Statistical significant differences by treatment did exist among students' scores on the computational skills

section of the experimenter-designed arithmetic achievement test. Since these differences favored the Textbook Treatment, the experimenter concluded that the Textbook Treatment had a significantly larger effect than the Laboratory Treatment upon computational skills of fourth grade children as measured by the experimenter-designed arithmetic achievement test.

3. No significant differences by treatment exist among students' scores on the concepts test of the Comprehensive Test of Basic Skills: Arithmetic Battery. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the knowledge of arithmetic concepts of fourth grade children, as measured by the Comprehensive Test of Basic Skills: Arithmetic Battery.

4. Statistical significant differences by treatment did exist among students' scores on the manipulation (concepts) section of the experimenter-designed arithmetic achievement test. However, the experimenter did not consider these differences to be educationally significant since the difference between mean scores by treatment was less than one point and less than one standard deviation. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had an educationally significantly larger effect than the other treatment upon the knowledge of arithmetic concepts of fourth grade children, as measured by the experimenter-designed arithmetic achievement test.

5. No significant differences by treatment exist among students' scores on the applications test of the Comprehensive Test of Basic Skills: Arithmetic Battery. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the ability of fourth grade children to apply arithmetic processes as measured by the Comprehensive Test of Basic Skills: Arithmetic Battery.

6. No significant differences by treatment exist among students' scores on the applications test of the experimenter designed arithmetic achievement test. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the ability of fourth grade children to apply arithmetic processes as measured by the experimenter-designed arithmetic achievement test.

7. No significant differences by treatment exist among students' scores on the Comprehensive Test of Basic Skills: Arithmetic Battery. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon arithmetic achievement of fourth grade children, as measured by the Comprehensive Test of Basic Skills: Arithmetic Battery.

8. No significant differences by treatment exist among students' scores on the experimenter-designed arithmetic achievement test. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon arithmetic achievement of fourth grade children, as measured by the experimenter-designed arithmetic achievement test.

9. Attitudes toward arithmetic of students receiving the Laboratory and Textbook Treatments did not differ significantly. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the attitudes of fourth grade children toward arithmetic, as measured by the Attitude Toward Arithmetic Scale.

10. Attitudes toward school of students receiving Laboratory and Textbook Treatments did not differ significantly. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the attitudes of fourth grade children toward school, as measured by the Attitude Toward School Scale.

11. Attitudes toward reading of students receiving the Laboratory and Textbook Treatments did differ significantly. Since the differences favored the Laboratory Treatment, the experimenter concluded that the Laboratory Treatment had a significantly larger effect than the Textbook Treatment upon the attitudes of fourth grade children toward reading, as measured by the Attitude Toward Reading Scale.

12. Attitudes toward spelling of students receiving the Laboratory and Textbook Treatments did not differ significantly. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the attitudes of fourth grade children toward spelling, as measured by the Attitude Toward Spelling Scale.

13. Attitudes toward science of students receiving the Laboratory and Textbook Treatments did not differ significantly. The experimenter concluded that neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon the attitudes of fourth grade children toward science, as measured by the Attitude Toward Science Scale.

14. School differences were significant on the Comprehensive Test of Basic Skills: Arithmetic Battery and each of its subtests: (1) computation, (2) concepts and (3) applications. The differences attributable to school effects were influenced by the instructional environment and teacher effectiveness occurring among the twelve schools involved in the study.

15. School differences were significant on the experimenter designed arithmetic achievement tests and each of its subtests: (1) manipulations (concepts), (2) computation, and (3) applications. The differences attributable to school effects were influenced by the instructional environment and teacher effectiveness occurring among the twelve schools involved in the study.

16. School differences were significant only on the attitude toward arithmetic scale. Attitudes of students toward arithmetic were influenced by the arithmetic instructional and school-neighborhood environment in which they study and live. School differences were not significant on the attitude toward school, reading, spelling and science scale.

17. Using the mean scores by treatment from attitudes scales employed in this experiment, the following rank order of attitudes from highest to lowest is obtained: (1) reading, (2) science, (3) spelling, (4) school and (5) arithmetic. Thus, students exhibit a less positive attitude toward arithmetic than for any other area studied. However, children exhibited relatively positive attitudes toward each of the five areas.

DISCUSSION OF THE STUDY

The purpose of the study was to determine the effects of two distinct diagnostic and remedial arithmetic programs upon the arithmetic achievement and attitude development of fourth grade children. Measures of children's achievement were obtained on their: (1)

knowledge of arithmetic concepts, (2) computational skills, and (3) problem solving ability. From the results of children's performance on two achievement tests, the experimenter concluded that: (1) neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon children's problem solving ability and knowledge of concepts and (2) the Textbook Treatment was more effective than the Laboratory Treatment for improving children's computational skills.

One reason for the significant difference between treatments in computational skills might be that teachers of the Laboratory Treatment stressed concept development to the extent that computational skills were not maintained and practiced. Further, children may have been engaged in the manipulation of objects illustrating arithmetic concepts and did not sufficiently develop and practice computational skills related to the concepts. Additional research must be conducted to determine the specific reasons for the significant difference in computational skills.

Furthermore, on the standardized arithmetic achievement test, the mean scores by treatment favor the Textbook Treatment in all three areas of arithmetic achievement. However, on the experimenter-designed arithmetic achievement test, the mean scores by treatments favor the Laboratory Treatment on the concepts and applications subtests while the mean scores favor the Textbook Treatment on the computational skills subtest. This apparent contradiction of results on the experimenter-designed and commercial subtests of concepts and applications may be resolved by considering the nature of each treatment and the exercises on each subtest. For instance, the experimenter-designed subtests contain exercises which were very similar to the instructional activities used in the Laboratory Treatment. In contrast to this, the commercial subtests contained exercises which were similar to the paper-pencil activities that were performed by children in the Textbook Treatment.

Moreover, examination of mean scores by treatments for the Comprehensive Test of Basic Skills: Arithmetic, Form Q1 (pre-test) and Form R1 (post-test) reveals that both treatments made about the same numerical gain in achievement for each of the three subtests. The differences between mean scores from initiation to termination of the study in three araas of arithmetic achievement for both treatments are given below:

Area	Laboratory	Textbook
Computational Skills	7.01	7.00
Concepts	1.98	1.69
Applications	0.57	0.92
Total	9.56	9.61

Thus, it is apparent that children in both treatments increased their arithmetic achievement, as measured by the Comprehensive Test of Basic Skills: Arithmetic, Forms Q1 and R1. However, for each of the three areas of arithmetic achievement, the difference between gain scores for the two treatments was not educationally significant.

On the other hand, the experimenter believes that children receiving both treatments would make larger gains in achievement if the instructional phase continued for a longer period. If this occurred, the difference between gain scores by treatments might be much larger.

Equally important is the second purpose of the study which was to determine the effects of two diagnostic and remedial arithmetic programs upon the attitude development of fourth grade children. The study obtained measurements of children's attitudes toward: (1) arithmetic, (2) school, (3) reading, (4) spelling and (5) science. From the results of children's performance on attitude scales, the experimenter concluded that: (1) neither diagnostic and remedial arithmetic treatment had a significantly larger effect than the other treatment upon children's attitudes toward arithmetic, school, spelling, and science and (2) the Laboratory Treatment had a significantly larger effect than the other treatment upon children's attitudes toward reading.

Further, examination of the mean scores by treatment reveals that the Laboratory Treatment has the larger mean score for all five of the areas included in the attitude scale. Thus, children receiving the Laboratory Treatment exhibited the more positive attitude of the two treatments, as measured by the experimenter-designed attitude scales.

In addition, children in both treatments exhibited very positive attitudes toward arithmetic, school, reading, spelling and science. Positive attitudes were exhibited even though the children: (1) have learning difficulties in arithmetic, (2) received instruction involving activities and skills which are often performed by children in lower grades, or (3) used a textbook designed for a lower grade. The experimenter believes that these positive attitudes were a result of children receiving an individualized arithmetic program, successfully solving arithmetic problems, and making progress in their study of arithmetic.

An integral role is played by the diagnostic procedures employed in this study, especially with the Laboratory Treatment. Sophisticated diagnostic procedures were successfully used by the teachers. Through the use of these diagnostic procedures, teachers of the Laboratory Treatment were able to locate specific arithmetic skills that were causing children difficulties. Furthermore, some arithmetic skills causing children difficulties, which are often missed by commerical diagnostic arithmetic tests, were made evident by the experimenter-designed diagnostic arithmetic test.

IMPLICATIONS OF THE STUDY

Some implications regarding diagnostic and remedial arithmetic programs at the fourth grade level were found in this study. These implications may serve as guides to people interested in diagnostic and remedial arithmetic programs for fourth grade children. Major implications have been grouped into sub-sections.

Diagnostic Procedures in Arithmetic

By contrasting children's performance on both the commercial and experimenter-designed diagnostic arithmetic tests, one can find additional information on the experimenter-designed diagnostic arithmetic test that enabled teachers to determine specific skills which are causing a child difficulties. Also, teachers were able to identify skills that are causing difficulties from a child's performance on the experimenter-designed diagnostic arithmetic test which were not apparent on the standardized diagnostic arithmetic test. Therefore, the experimenter recommends that diagnostic procedures employ interview techniques structured by arithmetic exercises which measure knowledge of skills identified in a hierarchy of subordinate skills and which require the child: (1) to manipulate objects illustrating a mathematical principle or number sentence, (2) to write a number sentence describing the action performed by an examiner on physical objects and (3) to compute answers to problems using paper and pencil.

Effect of Diagnostic and Remedial Programs on Achievement

Both diagnostic and remedial arithmetic treatments employed in this study enabled children to increase their achievement in arithmetic. The instructional phase of the study consisted of twenty-nine class periods. The experimenter believes that diagnostic and remedial arithmetic programs conducted for longer periods, such as a semester or a year, will result in much larger increases in arithmetic achievement than found in this study. Also, the experimenter feels that diagnostic and remedial arithmetic programs began in lower grade levels, such as grade two or three, will greatly increase children's arithmetic achievement.

Effect of Diagnostic and Remedial Programs on Attitudes

Separating children from their peers for special instruction in arithmetic, studying skills and doing activities often reserved for lower grade levels, and using an arithmetic textbook designated for a lower grade did not cause children in this study to develop negative attitudes toward arithmetic, school, reading, spelling or science. Thus, the experimenter concludes that diagnostic and remedial arithmetic programs for fourth grade children will not necessarily cause children to develop a negative attitude toward arithmetic, school, reading, spelling or science.

Effect of Diagnostic and Remedial Programs on Curriculum

The remedial arithmetic program for a child participating in this study was based upon his performance on a diagnostic test. Furthermore, teachers of the Laboratory Treatment designed activities for a specific skill and child. In essence, this approach resulted in an individualized arithmetic program for each child. Children engaged in an individualized arithmetic program were able to increase their arithmetic achievement. Therefore, the experimenter suggests that teachers use a child's performance on a diagnostic arithmetic test as a basis for designing an individualized arithmetic program to meet the child's particular needs.

SUGGESTIONS FOR FUTURE RESEARCH

Conclusions made in this study are applicable to the fourth grade population from which the children participating in this experiment were selected. Children involved in this experiment were below grade level in their arithmetic achievement. Educational opinion suggests that fourth grade students who are at or above grade level in their arithmetic achievement may profit from a diagnostic and individualized arithmetic program. Research to determine the effects of diagnostic and individualized arithmetic program upon fourth grade children who are at or above grade level in arithmetic achievement may produce results which are different from those obtained in this experiment.

Effects of diagnostic and remedial arithmetic programs upon attitudes and arithmetic achievement of children with arithmetic learning difficulties in the third, fifth or sixth grade were not determined by this study. Results from diagnostic and remedial arithmetic programs conducted at either the third, fifth, or sixth grade might differ from those obtained in this study.

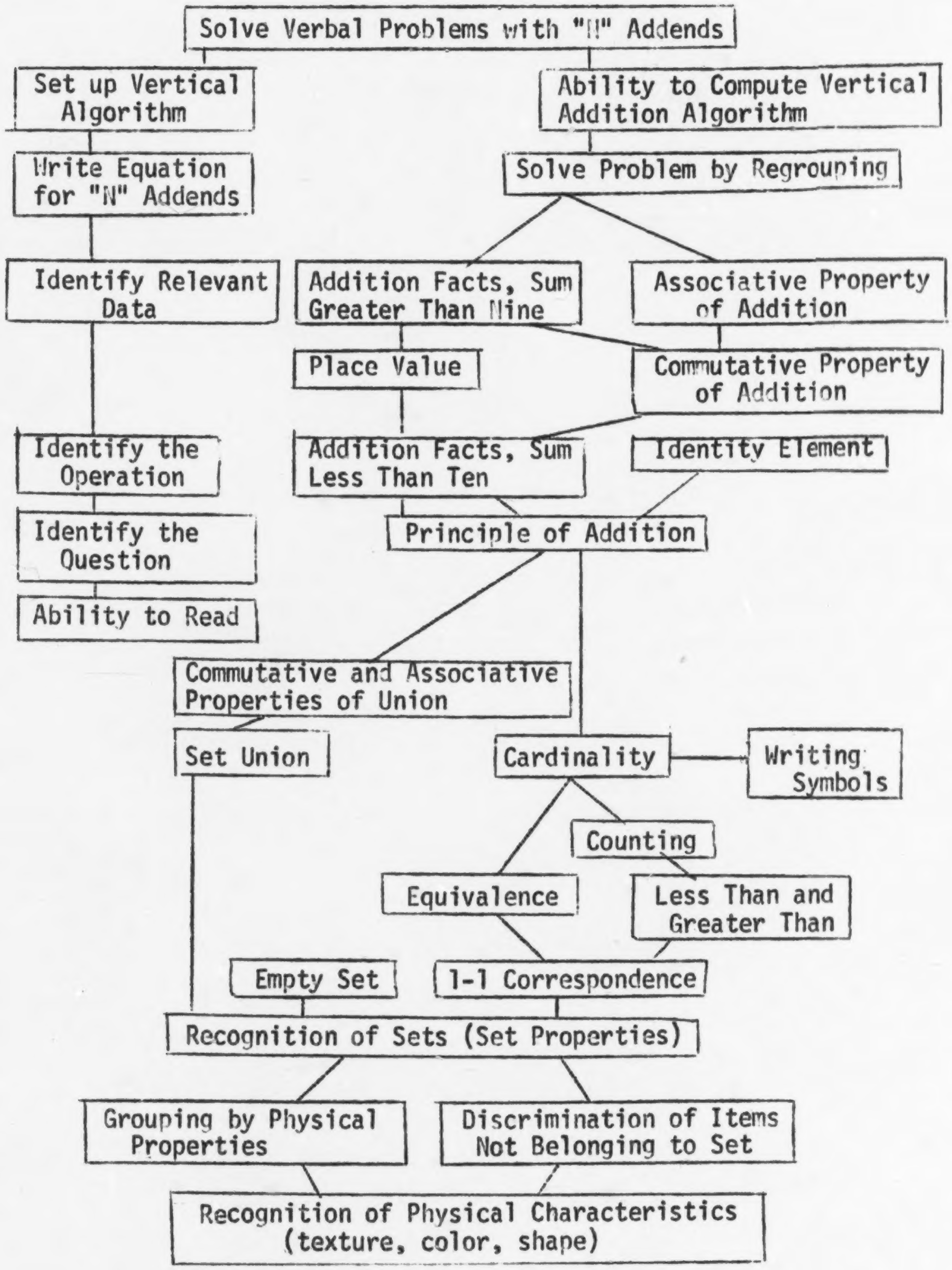
Conclusions formulated in this experiment are based upon a study of the effects of diagnostic and remedial arithmetic programs on arithmetic achievement for an instructional phase lasting seven weeks. The effects of a study conducted for longer periods of time may yield results which are different from those obtained in this experiment.

Conclusions regarding attitudes of children participating in this study were based upon results obtained from paper and pencil scales. Additional information regarding the effects of diagnostic and remedial arithmetic programs upon attitudes might have been obtained if interview techniques structured by specific questions or scales had been used.

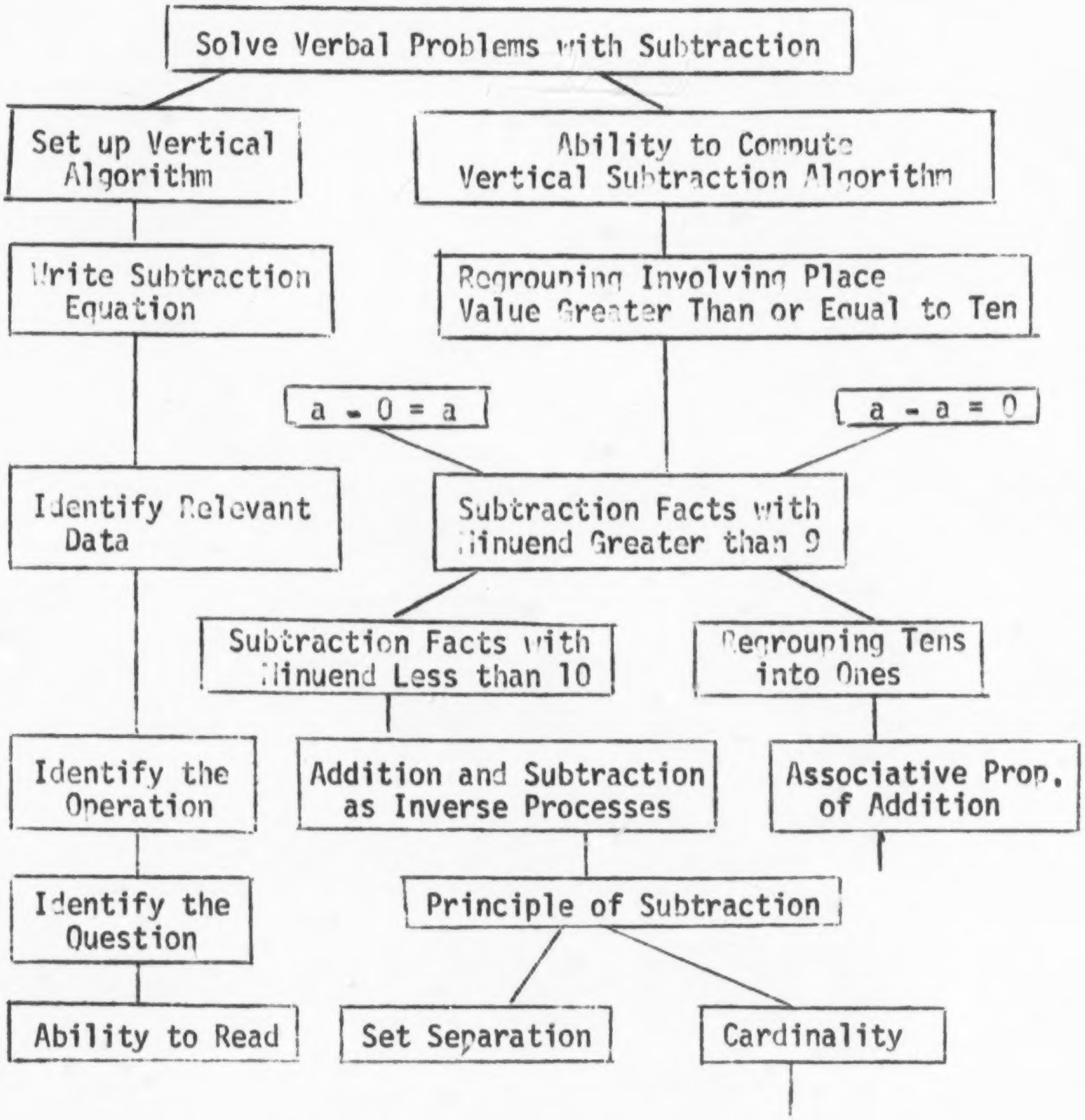
APPENDIX A

1. Hierarchy of Addition Skills
2. Hierarchy of Subtraction Skills
3. Hierarchy of Multiplication Skills
4. Hierarchy of Division Skills
5. Composite Hierarchy of Arithmetic Skills

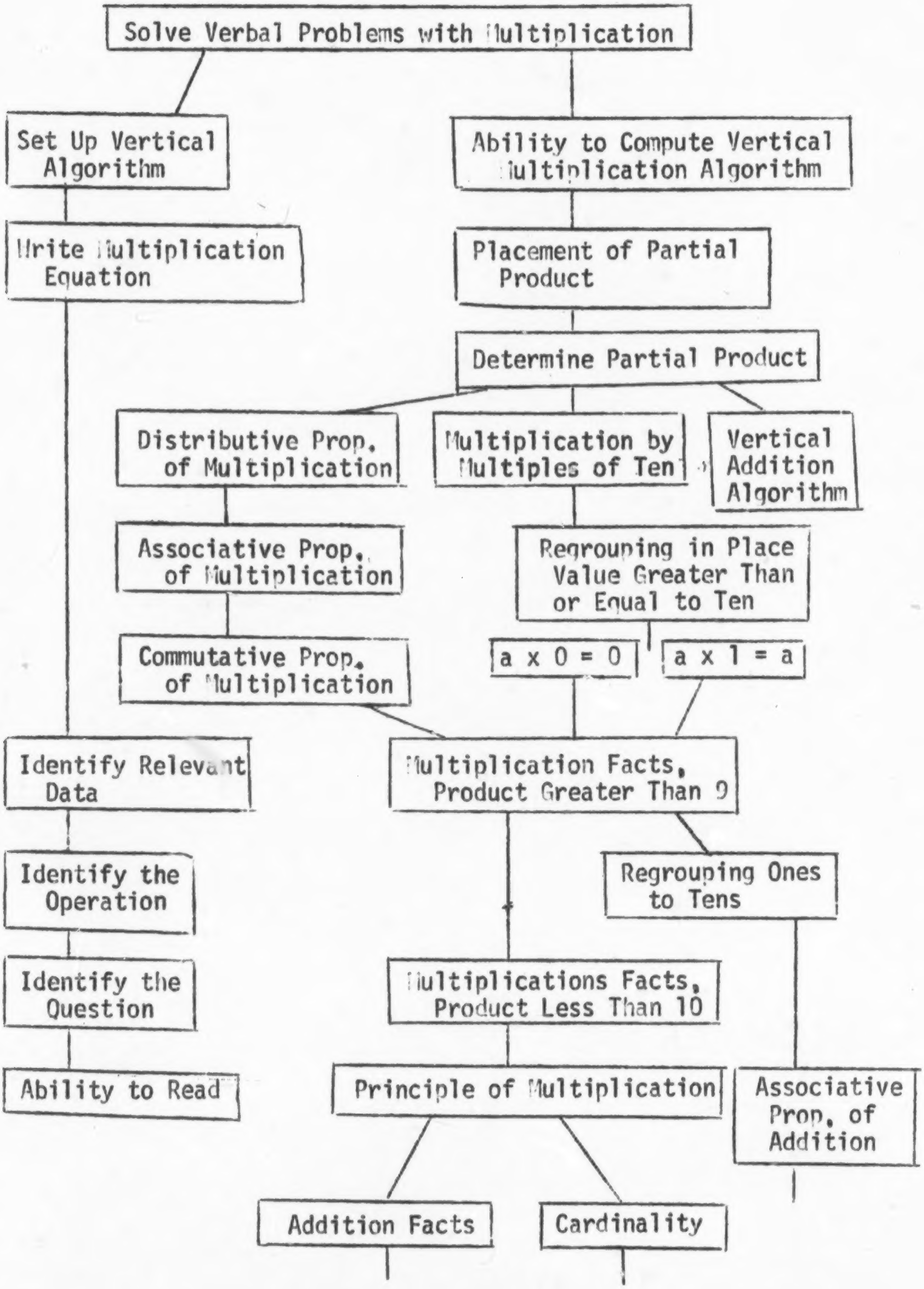
HIERARCHY OF ADDITION SKILLS


Solve Verbal Problems with "N" Addends
Set up Vertical Algorithm
Ability to Compute Vertical Addition Algorithm
Write Equation for "N" Addends
Solve Problem by Regrouping
Identify Relevant Data
Addition Facts, Sum Greater Than Nine
Associative Property of Addition
Place Value
Commutative Property of Addition
Identify the Operation
Addition Facts, Sum Less Than Ten
Identity Element
Identify the Question
Principle of Addition
Ability to Read
Commutative and Associative Properties of Union
Set Union
Cardinality
Writing Symbols
Counting
Equivalence
Less Than and Greater Than
Empty Set
1-1 Correspondence
Recognition of Sets (Set Properties)
Grouping by Physical Properties
Discrimination of Items Not Belonging to Set
Recognition of Physical Characteristics (texture, color, shape)

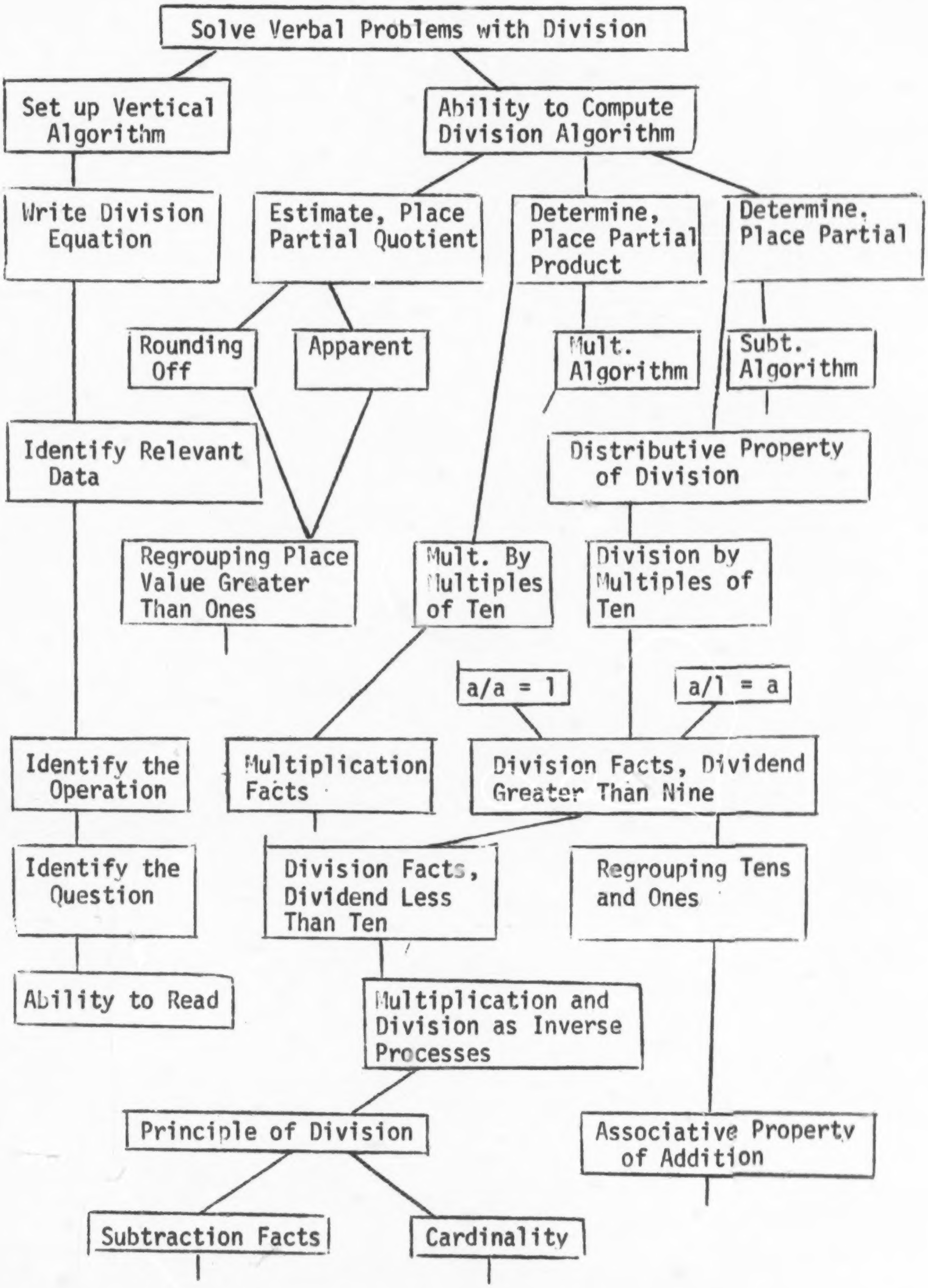
HIERARCHY OF SUBTRACTION SKILLS


Solve Verbal Problems with Subtraction
Set up Vertical Algorithm
Ability to Compute Vertical Subtraction Algorithm
Write Subtraction Equation
Regrouping Involving Place Value Greater Than or Equal to Ten
a - 0 = a
a - a = 0
Identify Relevant Data
Subtraction Facts with Minuend Greater than 9
Subtraction Facts with Minuend Less than 10
Regrouping Tens into Ones
Identify the Operation
Addition and Subtraction as Inverse Processes
Associative Prop. of Addition
Identify the Question
Principle of Subtraction
Ability to Read
Set Separation
Cardinality

HIERARCHY OF MULTIPLICATION SKILLS


Solve Verbal Problems with Multiplication
Set Up Vertical Algorithm
Ability to Compute Vertical Multiplication Algorithm
Write Multiplication Equation
Placement of Partial Product
Determine Partial Product
Distributive Prop. of Multiplication
Multiplication by Multiples of Ten
Vertical Addition Algorithm
Associative Prop. of Multiplication
Regrouping in Place Value Greater Than or Equal to Ten
Commutative Prop. of Multiplication
a x 0 = 0
a x 1 = a
Identify Relevant Data
Multiplication Facts, Product Greater Than 9
Identify the Operation
Regrouping Ones to Tens
Identify the Question
Multiplications Facts, Product Less Than 10
Ability to Read
Principle of Multiplication
Associative Prop. of Addition
Addition Facts
Cardinality

HIERARCHY OF DIVISION SKILLS


Solve Verbal Problems with Division
Set up Vertical Algorithm
Ability to Compute Division Algorithm
Write Division Equation
Estimate, Place Partial Quotient
Determine, Place Partial Product
Determine, Place Partial
Rounding Off
Apparent
Mult. Algorithm
Subt. Algorithm
Identify Relevant Data
Distributive Property of Division
Regrouping Place Value Greater Than Ones
Mult. By Multiples of Ten
Division by Multiples of Ten
a/a = 1
a/1 = a
Identify the Operation
Multiplication Facts
Division Facts, Dividend Greater Than Nine
Identify the Question
Division Facts, Dividend Less Than Ten
Regrouping Tens and Ones
Ability to Read
Multiplication and Division as Inverse Processes
Principle of Division
Associative Property of Addition
Subtraction Facts
Cardinality

COMPOSITE HIERARCHY

Division Algorithm

Determine, Place Partial Product   Determine, Place Partial Quotient   Determine, Place Partial Dividend

Rounding Off   Apparent

Multiplication Algorithm

Determine, Place Partial Product in Multiplication

Distributive Prop. of Multiplication   Mult. By Powers of Tens   Div. by Powers of Ten   Distributive Prop. of Division

Associative Property of Multiplication

Commutative Property of Multiplication

a x 0 = 0   a x 1 = a   a/a = 1   a/1 = a

Multiplication Facts   Division Facts

Addition Algorithm   Subtraction Algorithm

Regrouping Place Value Greater Than Ten

Associative Prop. of Add.   Mult. and Div. As Inverses

Commutative Prop. of Addition   Principle of Mult.   Principle of Division

a + 0 = a   a - a = 0   a - 0 = a

Addition Facts, Sum Greater Than Nine   Subtraction Facts, Minuend Greater Than Nine

Regrouping in Tens and Ones

Addition Facts, Sum Less Than Ten   Subtraction Facts, Minuend Less Than Ten

Addition and Subtraction as Inverses

Principle of Addition   Principle of Subtraction

Prop. of Set Union   Cardinality   Writing Symbols

Set Union   Counting

Equivalence   Less Than, Greater Than

Empty Set   1-1 Correspondence

Recognition of Sets (Set Properties)

Grouping by Physical Prop.   Discrimination of Items not in Set

Recognition of Physical Characteristics

APPENDIX B

1. Behavioral Objectives for Addition Skills
2. Behavioral Objectives for Subtraction Skills
3. Behavioral Objectives for Multiplication Skills
4. Behavioral Objectives for Division Skills

BEHAVIORAL OBJECTIVES
ADDITION

Solve Verbal Problems with "N" Addends

Given a set of fifteen one and two-step written problems involving "N" addends, the learner must do the following: (1) recognize those problems which involve additive situations, (2) correctly construct a given sentence for each additive problem, (3) set up a vertical addition algorithm, and (4) compute the correct answer to each of the problems with 95% of the problems computed correctly.

Set Up Vertical Addition Algorithm

Given ten addition number sentences involving sets of "N" addends with "N" digits, the learner must correctly align each addend into a vertical algorithm with 100% accuracy.

Write Mathematical Sentences for Addition Problems with "N" Addends

Given five written word and five orally presented addition problems involving "N" addends, the learner must identify the addends needed to correctly answer each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-verbal addition problems involving "N" addends, the learner must identify the addends needed to correctly answer each problem with 100% accuracy.

Identify the Operation of Addition

Given five written word and five non-verbal addition problems, the learner must specify that the operation of addition is necessary to identify the answers for each additive situation with 100% accuracy.

Identify the Question

Given five written word and five non-verbal addition problems involving "N" addends, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking me to answer?"

Ability to Read

(It is assumed by the researcher that each child at this point in the hierarchy must be able to read the printed words.)

Computing Vertical Addition Algorithm

When presented with fifteen vertical addition algorithms requiring both regrouping and non-regrouping and involving "N" addends that vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Solve Addition Problems by Regrouping (Carrying)

When presented with ten addition sentences involving "N" addends that require regrouping, the learner must be able to correctly compute each sum and explain what he has done with 100% accuracy.

Addition Facts Whose Sum is Greater Than Nine

A) Given rapid flash card presentations of forty-five addition facts whose sum is greater than nine but less than eighteen, the learner shall correctly call out each sum with 100% accuracy.

B) Given the forty-five written addition facts, the learner must correctly write each sum in 1 1/2 minutes or less with 100% accuracy.

Associative Property of Addition

When presented five situations with three or more sets of objects and shown how they may be grouped in more than one way without affecting the sum, the learner must correctly describe the grouping action by writing a number sentence with 100% accuracy.

Given sets of five written addition problems involving three or more addends, the learner must show that (a + b) + c = a + (b + c) by grouping three or more sets of objects to form two sets whose sum is equal to two sets grouped in another fashion with 100% accuracy.

Commutative Property of Addition

Given sets of ten written and five physically manipulated addition problems involving two addends, the learner must show that a+b=b+a with 100% accuracy.

Understanding Place Value

A) Given five sets of objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc. and write a numeral representing the composite number with 100% accuracy.

B) Given ten written numerals with varying place values represented, the learner must identify the name of and distinguish between the place values occupied by digits in the numerals with 100% accuracy.

C) Given five written numerals and/or five orally presented numbers, the learner must identify the respective numerals and/or numbers by showing the representations on either the abacus, the pocket chart or both with 100% accuracy.

Addition Facts Whose Sum is Less Than or Equal to Nine

A) Given rapid flash card presentations of the fifty-five addition facts whose sum is less than or equal to nine, the learner shall correctly call out each sum with 100% accuracy.

B) Given the fifty-five written addition facts, the learner must correctly write each sum in two minutes or less with 100% accuracy.

Identify Element

A) When presented with five written addition problems involving untaught numerals which are greater than ten, having zero as one of the addends, the learner must name the sum with 100% accuracy.

B) When presented with the number sentence a+0 (a is less than 10) and given a set of physical objects, the learner must correctly align the appropriate number of objects to form a set to represent the sum with 100% accuracy.

Principle of Addition

A) When presented with a series of five manipulations involving two or more disjoint sets of objects, the learner must correctly describe the action represented by writing appropriate number sentence for each with 100% accuracy.

B) When presented with five number sentences involving two or more addends each, the learner must correctly manipulate objects to demonstrate the action called for by each sentence with 100% accuracy.

Associative Property of Union

Given five manipulative problems involving the union of more than two disjoint sets of objects, such that (a+b)+c=a+(b+c), the learner must respond in each case to the question, "Does the arrangement of the sets of objects change the total?" by explaining that it does not with 100% accuracy.

Commutative Property of Union

Given five manipulative problems involving the union of two disjoint sets of objects, such that a+b=b+a, the learner must correctly respond in each case to the question, "Does changing the order of the objects change the total?" by explaining that it does not with 100% accuracy.

Set Union

A) When presented with sketches of five disjoint sets of objects and given the instructions to join the sets, the learner must draw sketches of the joined set in each of the five problems with 100% accuracy.

B) When presented with ten physical objects arranged into two or more disjoint sets, and told to join these objects to form one set, the learner must physically join each of the disjoint sets into a single set and identify the members of the new set by their characteristics with 100% accuracy.

Cardinality

To demonstrate his knowledge of cardinality, the child must do each of the following with 100% accuracy:

A) When rapidly exposed to flash cards demonstrating elementally grouped objects one - nine (exposure less than one second), the learner shall correctly call out the appropriate number represented by the card.

B) When exposed to nine ungrouped sets of physical and sketched objects, the learner must correctly write the numeral to represent the number of each set.

C) When randomly presented with the numerals 1-9, the learner must correctly sketch and/or manipulate objects to represent each numeral.

Write Meaningful Symbols for 0 - 9

When orally presented with the numbers 0 - 9 in random order, the learner will correctly write the appropriate numeral and word for each number.

Empty Set

When orally presented with five problems requiring him to count set members of empty sets, the learner shall correctly indicate that there are no objects in the set and call out the number "zero" to indicate the number of the set with 100% accuracy.

Meaningful Counting

A) Given five sets of objects of varying number, the learner must point to each object in turn while calling the appropriate number in sequence with 100% accuracy.

B) Given a set of discs with numerals 1 - 30 written on them, and given a set of 30 addition objects, the learner must match one-to-one and in an ordinal sequence the set of discs with the other set while calling the appropriate number with 100% accuracy.

Greater Than or Less Than

Given five written and five manipulative problem situations involving the representation of two or more sets of objects in each, the learner must identify those sets which are not equivalent to a given set and correctly identify whether they are greater than or less than the given set through one-to-one correspondence with 100% accuracy.

Equivalence

A) Given five problems involving two or more sets of various physical objects including two or more sets in each problem with the same cardinality, the learner must identify which two sets of each problem have the same number of objects in them by one-to-one correspondence with 100% accuracy.

One-To-One Correspondence

A) Given five situations involving two sets of objects the learner must physically match each object from one set with a corresponding object from the other set with 100% accuracy.

B) Given five worksheets with two sets of items shown on each, the learner must match the items in the one set with the other set by drawing lines between single items of each set with 100% accuracy.

Recognition of Sets

A) When presented with five sets of physical objects and five sets of sketched objects, the learner must orally list the characteristics of each set with 100% accuracy.

B) When presented with defined characteristics for each of five sets and given several sets of physical objects and several sets of sketched objects, the learner must identify each defined set by its specified characteristics with 100% accuracy.

Grouping by Physical Properties

Given five scrambled sets of physical objects, with each set having one or more characteristics in common only to itself when compared to other sets in the exercise, the learner must discriminate all objects with (a) given characteristics and classify them into sets as the teacher verbalizes the characteristics to look for with 100% accuracy.

Discrimination of Items Not Belonging to Set

Given five scrambled sets of physical objects, with each set having one or more characteristics in common only to itself when compared to other sets in the exercise, the learner must discriminate which objects do not belong in a particular set as the teacher verbalizes the characteristics of the set with 100% accuracy.

Recognition of Physical Characteristics

Given physical materials, the learner must describe with 100% accuracy the following characteristics as they occur: (a) color, (b) shape, (c) size, and (d) texture.

BEHAVIORAL OBJECTIVES
SUBTRACTION

Solve Verbal Problems With Subtraction

Given a set of fifteen one and two-step written problems involving at least two numbers, the learner must be able to do the following:

(1) recognize those problems which involve subtractive situations
(2) correctly construct a mathematical sentence for each subtractive problem
(3) set up a vertical subtraction algorithm
(4) compute the correct answer to each of the problems with 90% of the problems computed correctly.

Set Up Vertical Subtraction Algorithm

Given ten subtraction equations involving numerals with "N" digits, the learner must correctly align the minuend and subtrahend into a vertical algorithm with 100% accuracy.

Write Mathematical Sentences

Given five written word and five orally presented subtraction problems involving at least two numbers, the learner must correctly construct a subtraction equation for each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-verbal subtraction problems involving at least two numbers, the learner must identify the subtrahend and minuend needed to correctly answer each problem with 100% accuracy.

Identify the Operation

Given five written word and five non-verbal subtraction problems, the learner must specify that the operation of subtraction is necessary to identify the answers for each subtractive situation with 100% accuracy.

Identify the Question

Given five written word and five non-verbal subtraction problems involving at least two numbers, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking me to answer?"

Ability to Read

(It is assumed by the researcher that each child at this point in the hierarchy must be able to read the printed words.)

Computing Vertical Subtraction Algorithm

When presented with fifteen vertical subtraction algorithms requiring both regrouping and non-regrouping and involving two numerals that vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Regrouping Place Value Greater Than or Equal to Ten

A) Given five written numerals and five orally presented numbers, the learner must correctly write an equation using the associative property for addition to form a different written representation of each numeral with 100% accuracy.

B) Given five written numerals and five orally presented numbers, the learner must correctly show two different representations on the abacus for each numeral with 100% accuracy.

Subtraction of a Number From Itself

A) When presented with five written subtraction problems having the same number for minuend and subtrahend, the learner must name the difference with 100% accuracy.

B) When presented with the number sentence, a-a=?, and given a set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the difference with 100% accuracy.

Subtracting Zero

A) When presented with five written subtraction problems involving untaught numerals, having zero as the subtrahend, the learner must name the difference with 100% accuracy.

B) When presented with the number sentence, a - 0 = ?, and given a set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the difference with 100% accuracy.

Subtraction Facts Whose Minuend is Greater Than Nine

A) Given rapid flash card presentations of forty-five subtraction facts whose minuend is greater than nine but less than nineteen, the learner must correctly call out each difference with 100% accuracy.

B) Given the forty-five written subtraction facts, the learner must correctly write each difference in 1 1/2 minutes or less with 100% accuracy.

Regrouping Tens Into Ones

A) Given five sets of objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc. and write a numeral representing the composite number with 100% accuracy.

B) Given five situations in which objects are grouped into ones, tens, etc., the learner must regroup the objects into one less group of tens and ten more ones and correctly describe the action represented by writing appropriate number sentences for each with 100% accuracy.

C) Given five written numerals and five orally presented numbers, the learner must correctly write an equation showing the use of the associative property of addition to form a different written representation of each numeral with 100% accuracy.

Subtraction Facts Whose Minuend is Less Than Ten

A) Given rapid flash card presentations of fifty-five subtraction facts whose minuend is less than ten, the learner must correctly call out each difference with 100% accuracy.

B) Given the fifty-five written subtraction facts, the learner must correctly write each difference in 1 1/2 minutes or less with 100% accuracy.

Addition and Subtraction as Inverse Processes

Given ten problems, each involving two numbers, the learner will demonstrate with 100% accuracy the inverse relationship which exists between addition and subtraction by the following:

A) Write one addition equation and one subtraction equation which shows the undoing inherent to inverse processes for each pair of numbers.

B) Given one addition equation and one subtraction equation for each pair of numbers, the learner must manipulate objects to show the undoing process represented by the two equations.

Associative Property of Addition

A) When presented five situations with three or more sets of objects and shown how they may be grouped in more than one way without affecting the sum, the learner must correctly describe the grouping action by writing a number sentence with 100% accuracy.

B) Given sets of five written addition problems involving three or more addends, the learner must show that $(a+b)+c = a+(b+c)$ by grouping three or more sets of objects to form two sets whose sum is equal to two sets grouped in another fashion with 100% accuracy.

Principle of Subtraction

A) When presented with a series of five manipulations involving the separation of a set into two distinct sets, the learner must correctly describe the action represented by writing appropriate number sentences for each with 100% accuracy.

B) When presented with five number sentences involving subtraction, the learner must correctly manipulate objects to demonstrate the action called for by each sentence with 100% accuracy.

Set Separation

Given five sets of objects, the student physically separates each set of objects into two distinct subsets with 100% accuracy.

BEHAVIORAL OBJECTIVES
MULTIPLICATION

Solve Verbal Problems with Multiplication

Given a set of fifteen one and two-step written problems involving at least two factors, the learner must be able to do the following:

(1) recognize those problems which involve multiplicative situations
(2) correctly construct a mathematical sentence for each multiplicative problem
(3) set up a vertical multiplication algorithm
(4) compute the correct answer to each of the problems with 90% of the problems computed correctly.

Set Up Vertical Multiplication Algorithm

Given ten multiplication number sentences involving sets of two factors with "N" digits, the learner must correctly align each factor into a vertical algorithm with 100% accuracy.

Write Mathematical Sentences

Given five written word and five orally presented multiplication problems involving "N" factors, the learner must correctly construct a multiplication equation for each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-verbal multiplication problems involving "N" factors, the learner must identify the factors needed to correctly answer each problem with 100% accuracy.

Identify the Operation

Given five written word and five non-verbal multiplication problems, the learner must specify that the operation of multiplication is necessary to identify the answers for each multiplicative situation with 100% accuracy.

Identify the Questions

Given five written word and five non-verbal multiplication problems involving "N" factors, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking me to answer?"

Ability to Read

(It is assumed by the researcher that each child at this point in the hierarchy must be able to read the printed words.)

Computing Vertical Multiplication Algorithm

When presented with fifteen vertical multiplication algorithms requiring both regrouping and non-regrouping and whose factors vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Placement of Partial Product

When presented with ten vertical multiplication algorithms for which the partial products have been computed, the learner must correctly align the written partial products for each algorithm and explain what he has done with 100% accuracy.

Determine Partial Product

When presented with ten vertical multiplication algorithms requiring both regrouping and non-regrouping and involving two factors that vary in number of digits, the learner must correctly compute all of the partial products in each problem and explain what he has done with 100% accuracy.

Computing Vertical Addition Algorithm

When presented with fifteen vertical addition algorithms requiring both regrouping and non-regrouping and involving "N" addends that vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Multiplication By Powers of Ten

A) When presented with five written numerals and/or five orally presented numbers to be multiplied by powers of ten, the learner must correctly compute each product and explain what he has done with 100% accuracy.

B) When presented with five written untaught numerals and five orally presented untaught numbers to be multiplied by powers of ten, the learner must correctly compute each product by suffixing an appropriate number of zeros to each given numeral with 100% accuracy.

Distributive Property of Multiplication Over Addition

Given sets of five written and five physically manipulated multiplication problems involving two factors, one factor expressed as a sum of two addends, the learner must correctly write equations and manipulate objects with 100% accuracy to show that the product of the factors is equal to the sum of the products obtained from the factor times each addend.

Regrouping In Place Value Greater Than or Equal To Ten

A) Given five written numerals and five orally presented numbers, the learner must correctly write an equation using the associative property for addition to form a different written representation of each numeral with 100% accuracy.

B) Given five written numerals and five orally presented numbers, the learner must correctly show two different representations on the abacus for each numeral with 100% accuracy.

Associative Property of Multiplication

A) When presented with five situations involving sets of three factors, the learner must correctly write equations for each set of factors with 100% accuracy showing two different groupings of the factors and a product which is the same for each grouping.

B) Given sets of five written multiplication problems involving three factors whose product is less than 100, the learner must show that (axb) xc = ax (bxc) by physically manipulating objects into two equivalent sets with 100% accuracy.

Commutative Property of Multiplication

Given sets of ten written and five physically manipulated multiplication problems involving two factors, the learner must show that a x b = b x a with 100% accuracy.

Multiplication By One

A) When presented with five written multiplication problems involving untaught numerals, having one as a factor, the learner must name the product with 100% accuracy.

B) When presented with the number sentence, a x 1, and a given set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the product with 100% accuracy.

Multiplication By Zero

A) When presented with five written multiplication problems involving untaught numerals, having zero as one of the factors, the learner must name the product with 100% accuracy.

B) When presented with the number sentence, a x 0, and a given set of physical objects, the learner must correctly group the appropriate number of objects to form a set to represent the product with 100% accuracy.

Multiplication Facts Whose Products Are Greater Than Nine

A) Given rapid flash card presentations of the fifty-eight multiplication facts whose products are greater than nine, the learner shall correctly call out each product with 100% accuracy.

B) Given the fifty-eight written multiplication facts, the learner must correctly write each product in two minutes or less with 100% accuracy.

Regrouping Ones to Tens

A) Given five sets of objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc. and write a numeral representing the composite number with 100% accuracy.

B) Given five situations in which objects are grouped into ones, tens, etc. the learner must regroup the objects into one less group of tens and ten more ones and correctly describe the action represented by writing appropriate number sentences for each with 100% accuracy.

C) Given five written numerals and/or five orally presented numbers, the learner must correctly write an equation showing the use of associative property of addition to form a different written representation of each numeral with 100% accuracy.

Associative Property of Addition

A) When presented five situations with three or more sets of objects and shown how they may be grouped in more than one way without affecting the sum, the learner must correctly describe the grouping action by writing a number sentence with 100% accuracy.

B) Given sets of five written addition problems involving three or more addends, the learner must show that $(a + b) + c = a + (b + c)$ by grouping three or more sets of objects to form two sets whose sum is equal to two sets grouped in another fashion with 100% accuracy.

Multiplication Facts Whose Product Is Less Than Ten

A) Given rapid flash card presentations of the forty-two multiplication facts whose product is less than ten, the learner must correctly call out each product with 100% accuracy.

B) Given the forty-two written multiplication facts, the learner must correctly write each product in two minutes or less with 100% accuracy.

Principle of Multiplication

A) When presented with a series of five manipulations involving the joining of equivalent sets, the learner must correctly describe the action represented by writing appropriate number sentences for each with 100% accuracy.

B) When presented with five number sentences involving two factors each, the learner must correctly manipulate objects to demonstrate the action called for by each sentence with 100% accuracy.

Addition Facts

A) Given rapid flash card presentations of the 100 addition facts, the learner must correctly call out each sum with 100% accuracy.

B) Given the 100 written addition facts, the learner must correctly write each sum in two minutes or less with 100% accuracy.

BEHAVIORAL OBJECTIVES
DIVISION

Solve Verbal Problems With Division

Given a set of fifteen one and two-step written problems involving at least two numbers, the learner must be able to do the following:

(1) recognize those problems which involve division situations
(2) correctly construct a mathematical sentence for each division problem
(3) set up a vertical division algorithm
(4) compute the correct answer to each of the problems with 90% of the problems computed correctly.

Set Up Vertical Division Algorithm

Given ten division number sentences involving at least two numerals with "N" digits, the learner must correctly place the dividend and divisor into a vertical algorithm with 100% accuracy.

Write Mathematical Sentences

Given five written word and five orally presented division problems involving at least two numbers, the learner must correctly construct a division equation for each problem with 100% accuracy.

Identify Relevant Data

Given five written word and five non-verbal division problems involving at least two numbers, the learner must identify the dividend and divisor needed to correctly answer each problem with 100% accuracy.

Identify the Operation

Given five written word and five non-verbal division problems, the learner must specify that the operation of division is necessary to identify the answers for each division situation with 100% accuracy.

Identify the Question

Given five written word and five non-verbal division problems involving at least two numbers, the learner must answer the following question for each problem with 100% accuracy: "What is the question this problem is asking me to answer?"

Ability to Read

(It is assumed by the researcher that each child at this point in the hierarchy must be able to read the printed words.)

Ability to Compute Classical Division Algorithm

When presented with ten classical division algorithms involving divisor and dividend that vary in number of digits, the learner must compute the algorithms with 95% accuracy.

Estimating and Placing Partial Quotient

When presented with ten division problems, the learner must correctly estimate and place partial quotients and explain what he has done with 100% accuracy.

Determine and Place Partial Product

When presented with ten division problems, the learner must correctly compute and place the partial products in each problem with 100% accuracy by applying the vertical multiplication algorithm to the partial quotient times the divisor.

Apparent

When presented with ten numerals and instructions to represent each numeral in a form appropriate for the apparent method of estimating quotients, the learner must correctly write for each numeral the first digit of the numeral followed by zeros in lower place value positions.

Rounding Off

When presented with ten numbers, the learner must correctly round off each number to the nearest specified place value position with 100% accuracy.

Computing Vertical Subtraction Algorithm

When presented with ten vertical subtraction algorithms requiring both regrouping and non-regrouping and involving two factors that vary in number of digits, the learner must compute the difference with 95% accuracy.

Distributive Property of Division

Given sets of five written and five physically manipulated division problems, dividend written as sum of two or more addends, the learner must correctly write equations and manipulate objects with 100% accuracy to show that the quotient of the two numbers is equal to the sum of the quotients obtained from dividing each addend by the divisor.

Division by Powers of Ten

A) When presented with five written numerals and five orally presented numbers to be divided by powers of ten, the learner must correctly compute the quotient and explain what he has done with 100% accuracy.

B) When presented with five written untaught numerals and five orally presented untaught numbers to be divided by powers of ten, the learner must correctly compute each quotient by removing the appropriate number of zeros from each given numeral with 100% accuracy.

Multiplication by Powers of Ten

A) When presented with five written numerals and five orally presented numbers to be multiplied by powers of ten, the learner must correctly compute each product and explain what he has done with 100% accuracy.

B) When presented with five written untaught numerals and five orally presented untaught numbers to be multiplied by powers of ten, the learner must correctly compute each product by suffixing appropriate number of zeros to each given numeral with 100% accuracy.

Regrouping Place Value Greater Than or Equal to Ten

A) Given five written numerals and five orally presented numbers, the learner must correctly write an equation using the associative property for addition to form a different written representation of each numeral with 100% accuracy.

B) Given five written numerals and five orally presented numbers, the learner must correctly show two different written representations on the abacus for each numeral with 100% accuracy.

A Number Divided by One

A) When presented with five written division problems involving untaught numerals, having one as the divisor, the learner must name the quotient with 100% accuracy.

B) When presented with number sentence, a ÷ 1, and a given set of physical objects, the learner must correctly align the appropriate number of objects to form a set to represent the quotient with 100% accuracy.

A Number Divided by Itself

A) When presented with five written division problems involving untaught numerals, having the same numeral for dividend and divisor, the learner must name the quotient with 100% accuracy.

B) When presented with the number sentence, a ÷ a, a given set of physical objects, the learner must correctly separate the objects into equivalent sets to represent the quotient with 100% accuracy.

Multiplication Facts

A) Given rapid flash card presentations of the 100 multiplication facts, the learner shall correctly call out each product with 100% accuracy.

B) Given the 100 written multiplication facts, the learner must correctly write each product in two minutes or less with 100% accuracy.

Division Facts Whose Dividend is Greater Than Nine

A) Given rapid flash card presentations of the fifty-eight division facts whose dividend is greater than nine, the learner shall correctly call out each quotient with 100% accuracy.

B) Given the fifty-eight written division facts, the learner must correctly write each quotient in 1 1/2 minutes or less with 100% accuracy.

Regrouping Tens to Ones

A) Given five sets of objects whose cardinality is greater than ten, the learner must align the objects into groups of ones, tens, etc. and write a numeral representing the composite number with 100% accuracy.

B) Given five situations in which objects are grouped into ones, tens, etc., the learner must regroup the objects into one less group of tens and ten more ones and correctly describe the action represented by writing appropriate number sentence for each with 100% accuracy.

C) Given five written numerals and five orally presented numbers, the learner must correctly write an equation showing the use of associative property of addition to form a different written representation of each numeral with 100% accuracy.

Associative Property of Addition

A) When presented five situations with three or more sets of objects and shown how they may be grouped in more than one way without affecting the sum, the learner must correctly describe the grouping action by writing a number sentence with 100% accuracy.

B) Given sets of five written addition problems involving three or more addends, the learner must show that (a+b)+c = a+(b+c) by grouping three or more sets of objects to form two sets whose sum is equal to two sets grouped in another fashion with 100% accuracy.

Division Facts Whose Dividend is Less Than Ten

A) Given rapid flash card presentations of the thirty-two division facts whose dividend is less than ten, the learner shall correctly call out quotient with 100% accuracy.

B) Given the thirty-two written division facts, the learner must correctly write each quotient in one minute or less with 100% accuracy.

Multiplication and Division as Inverse Processes

Given ten problems each involving two numbers, the learner will demonstrate with 100% accuracy the inverse relationship which exists between multiplication and division by the following:

A) Write one multiplication equation and one division equation which shows the undoing inherent to inverse processes for each pair of numbers.

B) Given one multiplication equation and one division equation for each pair of numbers, the learner will manipulate objects to show the undoing process represented by the two equations.

Principle of Division

A) When presented with a series of five manipulations involving the separation of a set into equivalent sets, the learner must correctl describe the action represented by writing appropriate number sentences for each with 100% accuracy.

B) When presented with five number sentences involving division, the learner must correctly manipulate objects to demonstrate the action called for by each sentence with 100% accuracy.

Subtraction Facts

A) Given rapid flash card presentations of the 100 subtraction facts, the learner shall correctly call out each difference with 100% accuracy.

B) Given the 100 written subtraction facts, the learner must correctly write each difference in two minutes or less with 100% accuracy.

APPENDIX C

Experimenter-Constructed Evaluative Instruments

1. Revised Diagnostic Arithmetic Test
    - A. Teacher's Guide
    - B. Student Booklet
2. Arithmetic Achievement Test
    - A. Teacher's Guide
    - B. Student's Booklet

TEACHER'S GUIDE

EXPERIMENTER-DESIGNED DIAGNOSTIC TEST

PRINCIPLE OF ADDITION

A) Given: Teacher physically performs the joining action in the problem.

Directions: "I am going to use objects to show a number sentence. Write a number sentence to show what I have done."

1. Join a set of 5 objects with a set of 3 objects. (5 + 3 = 8)

B) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

2. 3 + 4 = /___/ (7)

IDENTITY ELEMENT

A) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

3. 8 + 0 = /___/ (8)

B) Given: Problems on a worksheet

Directions: "Write the answer to each problem."

4. 96 + 0 = /___/ (96)
5. /______/ = 1289 + 0 (1289)
6. 12485 + /______/ = 12485 (0)

ADDITION FACTS WHOSE SUM IS LESS THAN TEN

Given: Worksheet with 20 addition facts satisfying the specification identified above.

Directions: "Work as quickly as you can. You may work for one minute."

REGROUPING ONES AND TENS

A) Given: 24 ungrouped objects are placed by the teacher before the child.

Directions: "Put the objects into groups of ten. How many ones are left over? Write a numeral showing the number of tens and the number of ones."

8. 24 objects (~~//////////~~ ~~//////////~~ ////)
10 10 4

B) Given: Problems on a worksheet

Directions: "Write a number sentence showing the numeral rewritten."

9. 2 tens + 14 ones (2 tens + 14 ones = 34)

10. 12 ones + 3 tens (12 ones + 3 tens = 32)

11. 4 tens + 19 ones (4 tens + 19 ones = 59)

C) Given: 32 objects grouped into 3 sets of ten and 2 ones.

Directions: "Regroup the objects so that there is one less group of ten. Write a number sentence to show what you did."

12. 32 (32 = 20 + 12 or
32 = 2 tens + 12 ones)

COMMUTATIVE PROPERTY

A) Given: 25 objects for the child

Directions: "Use the objects to show that the number sentence is correct."

13. 4 + 3 = 3 + 4

B) Given: Problems on a worksheet

Directions: "Answer the questions by making a circle around the correct answer."

14.	3 + 2 =	5 + 1	2 + 3 answer	4 + 4
15.	7 + 5 =	5 + 5	7 + 2	5 + 7 answer
16.	10 + 8 =	8 + 10 answer	9 + 10	8 + 11

ADDITION FACTS WHOSE SUM IS GREATER THAN NINE

Given: Worksheet with 25 addition facts satisfying the specification identified above.

Directions: "Work as quickly as you can. You may work for one minute."

ASSOCIATIVE PROPERTY OF ADDITION

A) Given: Sets of 4, 5, and 2 objects. Teacher manipulates the sets (4 + 5) + 2 and 4 + (5 + 2) to illustrate the associative property of addition.

Directions: "Write a number sentence to show what I have done."

18. Sets of 4, 5, and 2 objects

B) Given: 25 objects for the child.

Directions: "Use the objects to show that the number sentence is correct."

19. (2 + 3) + 4 = 2 + (3 + 4)

PLACE VALUE AND REGROUPING

A) Given: The specified numerals and place value position to be asked about.

Directions: "What is the place value occupied by _____?"

20. 5 in 75691	(thousands)
21. 4 in 96742	(tens)
22. 7 in 81763	(hundreds)

B) Given: Problems on a worksheet.

Directions: "Write a number sentence to show how numerals may be borrowed to form more ones, tens, etc."

23. 56 with a regrouping of tens to ones
(56 = 40 + 16)

24. 475 with a regrouping of hundreds to tens
(475 = 300 + 170 + 5)

25. 3829 with a regrouping from thousands to hundreds
(3829 = 2000 + 1800 + 20 + 9

C) Given: Problems on a worksheet

Directions: "Find the answer to each problem below."

26. 70 + 18 (88)

27. 400 + 5 + 180 (585)

28. 4000 + 2600 + 40 + 9 (6649)

COMPUTING THE VERTICAL ADDITION ALGORITHM

Given: Worksheet containing the problems

Directions: "Find the answer for each problem."

(No regrouping required)

29. $\begin{array}{r} 35 \\ +24 \\ \hline \end{array}$ (59)

30. $\begin{array}{r} 6 \\ 21 \\ 132 \\ +\ 40 \\ \hline \end{array}$ (199)

31. $\begin{array}{r} 231 \\ 14 \\ 120 \\ +\ \ 4 \\ \hline \end{array}$ (369)

(Regrouping required)

32. $\begin{array}{r} 96 \\ +47 \\ \hline \end{array}$ (143)

33. $\begin{array}{r} 7 \\ 45 \\ 273 \\ +\ 95 \\ \hline \end{array}$ (420)

34. $\begin{array}{r} 456 \\ 78 \\ 592 \\ +\ \ 9 \\ \hline \end{array}$ (1135)

or

SET SEPARATION

Given: 15 objects for the child.

Directions: "Separate the fifteen objects into a set of 9 objects and a set of 6 objects."

1. 15 objects into a set of 9 objects and a set of 6 objects.

PRINCIPLE OF SUBTRACTION

A) Given: Teacher physically performs the separating action.

Directions: "Write a number sentence to show what I have done."

2. 8 take away 5 (8 - 5 = 3)

B) Given: Give the child 25 objects in two groups of ten and five ones.

Directions: "Using these objects, show how to find the answer to the number sentence."

3. /______/ = 13 - 8 (5)

ADDITION AND SUBTRACTION AS INVERSE PROCESSES

A) Given: Give the child 25 objects.

Directions: "Use the objects to show that addition is the opposite of subtraction."

4. 8 + 7 = 15 and 15 - 8 = 7

B) Given: Worksheet with a problem

Directions: "Use the seven and nine to write and solve one addition number sentence. Now use these numerals to write one subtraction number sentence."

5. 7 and 9 (7 + 9 = 16 16 - 9 = 7 or some equivalent form)

SUBTRACTION FACTS WITH MINUEND LESS THAN TEN

Given: Worksheet with 25 subtraction facts satisfying the specification identified above.

Directions: "Work as quickly as you can. You may work for one minute."

SUBTRACTION FACTS WITH MINUEND GREATER THAN NINE

Given: Worksheet with 25 subtraction facts satisfying the specification identified above.

Directions: "Work as quickly as you can. You may work for one minute."

SUBTRACTING ZERO

A) Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to each number sentence."

8. 9 - 0 = □ (9)

B) Given: Problems on a worksheet

Directions: "Write the answer to each problem."

9. 96 - 0 = □ (96)

10. □ = 1289 - 0 (1289)

11. 4569 - □ = 4569 (0)

SUBTRACTING A NUMBER FROM ITSELF

A) Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to each number sentence."

12. 5 - 5 □ (0)

13. □ = 13 - 13 (0)

B) Given: Problems on a worksheet.

Directions: "Write the answer to each problem."

14. 98 - 98 = □ (0)

15. □ = 1287 - 1287 (0)

16. 4569 - □ = 0 (4569)

COMPUTING THE VERTICAL SUBTRACTION ALGORITHM

Given: Problems on a worksheet.

Directions: "Find the answer to each problem."

(No regrouping)

$$17.\quad \begin{array}{r} 48 \\ -32 \\ \hline \end{array} \qquad 18.\quad \begin{array}{r} 476 \\ -235 \\ \hline \end{array} \qquad 19.\quad \begin{array}{r} 9876 \\ -5431 \\ \hline \end{array}$$

(16) (241) (4445)

(Regrouping)

$$20.\quad \begin{array}{r} 987 \\ -258 \\ \hline \end{array} \qquad 21.\quad \begin{array}{r} 502 \\ -357 \\ \hline \end{array} \qquad 22.\quad \begin{array}{r} 2457 \\ -789 \\ \hline \end{array}$$

(729) (145) (1668)

PRINCIPLE OF MUTLIPLICATION

A) Given: Teacher physically performs the joining action in each situation.

Directions: "Write an addition sentence to show what I have done. Now, write a multiplication number sentence."

1. Join 2 equivalent sets with 3 objects in each set.
(3 + 3 = 6 2 x 3 = 6)

2. Join 5 equivalent sets with 2 objects in each set.
(2 + 2 + 2 + 2 + 2 = 10 5 x 2 = 10)

B) Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to each number sentence."

3. 3 x 2 = ▱ (6)

4. ▱ = 4 x 5 (20)

MULTIPLICATION FACTS

A) Given: Worksheet with 25 multiplication facts involving a factor less than or equal to five.

Directions: "Work as quickly as you can. You may work for one minute."

B) Given: Worksheet with 25 multiplication facts involving a factor greater than 5 but less than ten.

Directions: "Work as quickly as you can. You may work for one minute."

MULTIPLICATION BY ZERO

A) Given: 25 objects for the child.

Directions: "Using these objects, show how to find the answer to the number sentence."

7. 3 x 0 = ▱ (0)

B) Given: Worksheet with problems.

Directions: "Write the answer to each problem."

8. 98 x 0 = /   / (0)

9. /   / = 0 x 7869 (0)

10. 1289 x /   / = 0 (0)

MULTIPLICATION BY ONE

A) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

11. 5 x 1 = /   / (5)

12. 12 x /   / = 12 (1)

B) Given: Worksheet with problems

Directions: "Write the answer to each problem."

13. 97 x 1 = /   / (97)

14. 7891 x /   / = 7891 (1)

COMMUTATIVE PROPERTY OF MULTIPLICATION

A) Given: 25 objects for the child

Directions: "Use the objects to show that the number sentence is correct."

15. 2 x 3 = 3 x 2

B) Given: Worksheet with problems

Directions: "Answer the questions by making a circle around the correct answer."

16. 5 x 4 =	19	4 x 5 answer	7 x 3
17. 3 x 4 =	4 x 3 answer	13	3 x 5
18. 8 x 2 =	3 x 8	15	2 x 8 answer

ASSOCIATIVE PROPERTY OF MULTIPLICATION

Given: Set of three factors on a worksheet

Directions: "Multiply the three factors by grouping them in two different ways."

19. 4, 5, and 2 (4x5)x2 = 4x(5x2) = (4x2)x5

DISTRIBUTIVE PROPERTY OF MULTIPLICATION

A) Given: Worksheet of problems

Directions: "Finish each number sentence to find the answer."

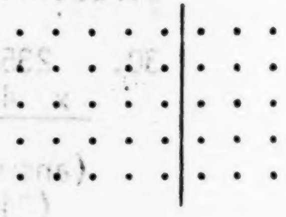
20. 5 x 7 = 5 x (4 + 3) = ▱ (20) + ▱ (15) = ▱ (35)

21. 6 x 9 = 6 x (5 + 4) = (30) + (24) = (54)

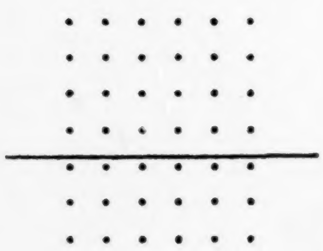
B) Given: Specified problems on a worksheet and an array

Directions: "Show the number sentence by drawing lines on the array."

22. 5 x 8 = 5 x (5 + 3) (answer)



23. 7 x 6 = (4 + 3) x 6 (answer)



MULTIPLICATION BY POWERS OF TEN

Given: Problems on a worksheet

Directions: "Write the answer to each problem."

24. 89 x 10 = / / (890)

25. 100 x 7896 = / / (789600)

26. 1289 x 1000 = / / (1289000)

DETERMINE AND PLACE THE PARTIAL PRODUCT

Given: Problems on a worksheet

Directions: "Find the part of the answer asked for in each problem below."

27. 56 (56 x 4)
x24
(224)

28. 1234 (1234 x 300)
x 319
(370200)

29. 542 (542 x 30)
x136
(16260)

VERTICAL MULTIPLICATION ALGORITHM

Given: Problems on a worksheet

Directions: "Find the answer for each problem."

30. 235
x 4
(answer)
(940)

31. 142
X 35
(answer)
710
426
4970

32. 405
x213
(answer)
1215
415
810
86265

PRINCIPLE OF DIVISION

A) Given: Teacher physically performs the separating action in each situation.

Directions: "Write a number sentence to show what I have done."

1. 12 objects separated into 4 equivalent sets
(12 ÷ 6 = 4)

2. 24 objects separated into equivalent sets containing 6 objects each
(24 ÷ 6 = 4)

B) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

3. 15 ÷ 5 = /____/ (3)

DIVISION FACTS

A) Given: Worksheet with 25 division facts whose divisor is less than or equal to five

Directions: "Work as quickly as you can. You may work for one minute."

B) Given: Worksheet with 25 division facts whose divisor is greater than 5 but less than 10.

Directions: "Work as quickly as you can. You may work for one minute."

A NUMBER DIVIDED BY ITSELF

A) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

6. 5 ÷ 5 = /____/

B) Given: Problems on a worksheet

Directions: "Write the answer to each number sentence."

7. 96 ÷ 96 = /___/ (1)

8. /___/ = 4561 ÷ 4561 (1)

9. 12879 ÷ /___/ = 1 (12879)

A NUMBER DIVIDED BY ONE

A) Given: 25 objects for the child

Directions: "Using these objects, show how to find the answer to the number sentence."

10. 9 ÷ 1 = /___/ (9)

B) Given: Problems on a worksheet

Directions: "Write the answer to each number sentence."

11. 97 ÷ 1 = /___/ (97)

12. /___/ = 5697 ÷ 1 (5697)

13. 997 ÷ /___/ = 997 (1)

DIVISION BY POWERS OF TEN

Given: Problems on a worksheet

Directions: "Write the answer to each number sentence."

14. 790 ÷ 10 = /___/ (79)

15. 8900 ÷ 100 = /___/ (89)

16. /___/ = 78000 ÷ 1000 (78)

17. /___/ = 65000 ÷ 100 (650)

18. 42000 ÷ 10 = /___/ (4200)

MULTIPLICATION AND DIVISION AS INVERSE PROCESSES

A) Given: 25 objects for the child

Directions: "Use the objects to show that multiplication is the opposite of division."

19. 18 ÷ 6 = 3 6 x 3 = 18

B) Given: Worksheet with a pair of numerals

Directions: "Use the 5 and 6 to write and solve a multiplication number sentence. Now, use these numerals to write one division number sentence."

20. 5 and 6 (5 x 6 = 30 and 30 ÷ 5 = 6 or some equivalent form)

DISTRIBUTIVE PROPERTY OF DIVISION

Given: Problem on a worksheet

Directions: "Finish the number sentence to find the answer."

21. 24 ÷ 3 = (12 + 9 + 3) ÷ 3 = ▱ + ▱ + ▱ = ▱

ROUNDING OFF

Given: Problems on a worksheet

Directions: "Round off each numeral to the place value which I tell you." Teacher says "_____."

22.	54 nearest ten	(50)
23.	4561 nearest hundred	(4600)
24.	47499 nearest thousand	(47000)
25.	67951 nearest hundred	(68000)

DETERMINE THE PARTIAL DIVIDEND

Given: Problems on a worksheet

Directions: "Circle the numerals you divide first. For example, what does 78 go into first?"

26. 78/67951 (679)

27. 725/147867 (1478)

Directions: "Find the second numeral of the answer."

4
28. 53/24567
212

(6)

DETERMINE AND PLACE THE PARTIAL QUOTIENT

Given: Problems on a worksheet

Directions: "Find and place the first numeral in the answer."

(8)
29. 78/67951

(1)
30. 45/64789

Directions: "Find and place the second numeral in the answer."

8 (6)
31. 53/45678
424
327

DETERMINE AND PLACE THE PARTIAL PRODUCT

Given: Problems on a worksheet

Directions: "Do the next step in the problem."

4
32. 53/2547

(212)

24
33. 215/51692
430
869

(860)

846
34. 4/3387
32
18
16
27

(24)

VERTICAL DIVISION ALGORITHM

Given: Problems on a worksheet

Directions: "Find the answer."

35. 9/936

answer
104
9/936
9
36
36
0

36. 24/3567

answer
148
24/3567
24
116
96
207
192
15

STUDENT'S WORKSHEETS

DIAGNOSTIC INVENTORY

1.

2. 3 + 4 = ☐

---

3. 8 + 0 = ☐

4. 96 + 0 = ☐

5. ☐ = 1289 + 0

6. 12485 + ☐ = 12485

7.

8	5	2	1	5	3	
+1	+4	+4	+7	+2	+3	
5	6	4	6	1	2	1
+3	+1	+4	+3	+5	+3	+2
4	6	3	1	7	2	1
+1	+2	+4	+3	+2	+2	+1

8.

9. 2 tens + 14 ones

10. 12 ones + 3 tens

11. 4 tens + 19 ones

12.

---

13. 4 + 3 = 3 + 4

14. 3 + 2 =   5 + 1   2 + 3   4 + 7

15. 7 + 5 =   5 + 5   7 + 2   5 + 7

16. 10 + 8 =   8 + 10   9 + 9   8 + 11

17.	9 +1	8 +8	4 +7	2 +9	7 +7	3 +8	9 +3	5 +5	
	7 +3	4 +9	8 +4	5 +6	9 +5	6 +4	2 +8	7 +5	7 +8
	6 +9	5 +8	6 +6	7 +9	8 +6	6 +7	9 +8	9 +9	

18.

19. (2 + 3) + 4 = 2 + (3 + 4)

---

20. 5 in 75691

21. 4 in 96742

22. 7 in 81763

---

23. 56 with a regrouping of tens to ones

24. 475 with a regrouping of hundreds to tens

25. 3829 with a regrouping of thousands to hundreds

26. 70 + 18

27. 400 + 5 + 180

28. 4000 +2600 + 40 + 9

29.	35 +24	30.	6 21 132 + 40	31.	231 14 120 + 4
32.	96 +47	33.	7 45 273 + 95	34.	456 78 592 + 9

1.

2.

3. ▱ = 13 - 8

4. 8 + 7 = 15 15 - 8 = 7

5. 7 and 9

6.

$$\begin{array}{rrrrrrrr} 9 & 8 & 7 & 9 & 7 & 7 & 9 & 6 \\ \underline{-8} & \underline{-5} & \underline{-6} & \underline{-7} & \underline{-1} & \underline{-5} & \underline{-5} & \underline{-1} \end{array}$$

$$\begin{array}{rrrrrrrrr} 8 & 9 & 7 & 8 & 9 & 6 & 8 & 5 & 3 \\ \underline{-2} & \underline{-6} & \underline{-4} & \underline{-6} & \underline{-3} & \underline{-5} & \underline{-3} & \underline{-4} & \underline{-2} \end{array}$$

$$\begin{array}{rrrrrrrr} 9 & 8 & 6 & 9 & 6 & 8 & 7 & 5 \\ \underline{-1} & \underline{-4} & \underline{-3} & \underline{-2} & \underline{-4} & \underline{-7} & \underline{-2} & \underline{-2} \end{array}$$

7.

$$\begin{array}{rrrrrrrr} 18 & 16 & 13 & 14 & 14 & 17 & 14 & 13 \\ \underline{-\ 9} & \underline{-\ 9} & \underline{-\ 8} & \underline{-\ 9} & \underline{-\ 6} & \underline{-\ 9} & \underline{-\ 8} & \underline{-\ 7} \end{array}$$

$$\begin{array}{rrrrrrrrr} 11 & 15 & 16 & 12 & 11 & 17 & 14 & 15 & 14 \\ \underline{-\ 6} & \underline{-\ 9} & \underline{-\ 7} & \underline{-\ 5} & \underline{-\ 3} & \underline{-\ 8} & \underline{-\ 5} & \underline{-\ 8} & \underline{-\ 7} \end{array}$$

$$\begin{array}{rrrrrrrr} 15 & 13 & 12 & 12 & 13 & 15 & 11 & 13 \\ \underline{-\ 7} & \underline{-\ 6} & \underline{-\ 4} & \underline{-\ 7} & \underline{-\ 5} & \underline{-\ 6} & \underline{-\ 5} & \underline{-\ 4} \end{array}$$

8. 9 - 0 = ▱

9. 96 - 0 = ▱

10. ▱ = 1289 - 0

11. 4569 - ▱ = 4569

12. 5 - 5 = ☐

13. ☐ = 13 - 13

14. 98 - 98 = ☐

15. ☐ = 1287 - 1287

16. 4569 - ☐ = 0

---

17. $\begin{array}{r} 48 \\ -32 \\ \hline \end{array}$ 18. $\begin{array}{r} 476 \\ -235 \\ \hline \end{array}$ 19. $\begin{array}{r} 9876 \\ -5431 \\ \hline \end{array}$

20. $\begin{array}{r} 987 \\ -258 \\ \hline \end{array}$ 21. $\begin{array}{r} 502 \\ -357 \\ \hline \end{array}$ 22. $\begin{array}{r} 2457 \\ -\ 789 \\ \hline \end{array}$

1.

2.

3. 3 x 2 = ▱

4. ▱ = 4 x 5

5.	2	3	5	9	3	2	8	4	
	x2	x4	x3	x2	x6	x7	x2	x5	
	2	3	4	5	3	2	9	2	4
	x5	x2	x3	x5	x7	x6	x2	x3	x4
	3	4	3	5	3	5	3	2	
	x3	x2	x5	x2	x8	x4	x9	x4	

6.	6	7	8	9	4	5	9	5	
	x6	x8	x9	x7	x6	x7	x4	x6	
	6	7	8	9	4	5	7	9	5
	x7	x7	x6	x8	x7	x9	x9	x6	x8
	7	6	8	9	4	6	8	4	
	x6	x8	x8	x9	x8	x9	x7	x9	

---

7. 3 x 0 = ▱

8. 98 x 0 = ▱

9. ▱ = 0 x 7869

10. 1289 x ▱ = 0

---

11. 5 x 1 = /______/

12. 12 x /______/ = 12

13. 97 x 1 = /______/

14. 7891 x /______/ = 7891

---

15. 2 x 3 = 3 x 2

16. 5 x 4 = 19 4 x 5 7 x 3

17. 3 x 4 4 x 3 13 3 x 5

18. 8 x 2 = 3 x 8 15 2 x 8

---

19. 4, 5, and 2

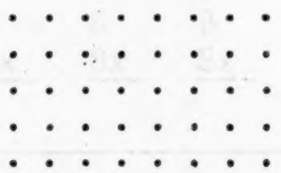
---

20. 5 x 7 = 5 x (4 + 3) = /______/ + /______/ = /______/

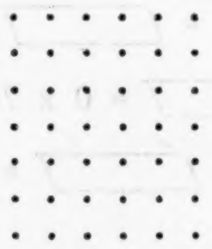
21. 6 x 9 = 6 x (5 + 4) =

---

22. 5 x 8 = 5 x (5 + 3)



23. 7 x 6 = (4 + 3) x 6



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24. 89 x 10 = ▱

25. 100 x 7896 = ▱

26. 1289 x 1000 = ▱

---

27. 56 (56 x 4)
x24

28. 1234 (1234 x 300)
x 319

29. 542 (542 x 30)
x136

---

30. 235
x 4

31. 142
x 35

32. 405
x213

1.

2.

3. 15 ÷ 5 = ▱

4. 4 ÷ 2
12 ÷ 3
20 ÷ 4
15 ÷ 5
18 ÷ 2
18 ÷ 3
14 ÷ 2
16 ÷ 2

10 ÷ 5
6 ÷ 3
12 ÷ 4
25 ÷ 5
21 ÷ 3
12 ÷ 2
18 ÷ 9
6 ÷ 2
16 ÷ 4

9 ÷ 3
8 ÷ 4
15 ÷ 3
10 ÷ 2
24 ÷ 3
20 ÷ 5
27 ÷ 3
8 ÷ 2

5. 36 ÷ 6
56 ÷ 7
72 ÷ 8
63 ÷ 9
24 ÷ 6
35 ÷ 7
36 ÷ 9
30 ÷ 6

42 ÷ 6
49 ÷ 7
48 ÷ 8
72 ÷ 9
28 ÷ 7
45 ÷ 9
63 ÷ 7
54 ÷ 9
32 ÷ 8

40 ÷ 8
42 ÷ 7
48 ÷ 6
64 ÷ 8
81 ÷ 9
54 ÷ 6
56 ÷ 8
36 ÷ 4

6. 5 ÷ 5 = ▱

7. 96 ÷ 96 = ▱

8. ▱ = 4561 ÷ 4561

9. 12879 ÷ ▱ = 1

10. 9 ÷ 1 = ▱

11. 97 ÷ 1 = ▱

12. ▱ = 5697 ÷ 1

13. 997 ÷ ▱ = 997

14. 790 ÷ 10 = ▱

15. 8900 ÷ 100 = ▱

16. ▱ = 78000 ÷ 1000

17. ▱ = 65000 ÷ 100

18. 42000 ÷ 10 = ▱

19. 18 ÷ 6 = 3 6 x 3 = 18

20. 5 and 6

21. 24 ÷ 3 = (12 + 9 + 3) ÷ 3 = ▱ + ▱ + ▱ = ▱

22. 54 nearest ten

23. 4561 nearest hundred

24. 47499 nearest thousand

25. 67951 nearest hundred

26. 78/ 67951

27. 725/ 147867

4
28. 53/ 24567
212

---

29. 78/ 67951

30. 45/ 64789

8
31. 53/ 45678
424
327

4
32. 53/ 2547

24
33. 215/ 51692
430
869

846
34. 4/ 3387
32
18
16
27

---

35. 9/ 936

36. 24/ 3567

TEACHER'S GUIDE

EXPERIMENTER-DESIGNED ACHIEVEMENT TEST

Directions:

A. Manipulation section may be given individually or to small groups. You decide which situation is best for you. Stay as close to the printed directions for each problem as possible.

B. Computation and word problems sections may be given as a group test. Children receiving the Textbook and Laboratory Treatments may be combined for the administration of these two sections. The computation and word problems sections should be administered in one sitting. A combined time of 40 minutes should be allowed for the administration of both sections.

MANIPULATION

GIVEN: Teacher physically performs the action in each situation.

DIRECTIONS: "I am going to use objects to show a number sentence. Write a number sentence to show what I have done."

PROBLEMS:

1. Teacher removes 6 objects from a set of 14 objects.
2. Teacher joins a set of 6 objects with a set of 4 objects.
3. Teacher separates a set of 12 objects into 3 sets of 4 objects each.
4. Teacher joins 2 sets of 3 objects in each set.
5. Teacher joins a set of 4 objects, a set of 2 objects, and a set of 5 objects.

GIVEN: 25 objects for the child. Teacher shows each number sentence to the child.

DIRECTIONS: "Using these objects, show how to find the answer to this number sentence."

PROBLEMS:

6. Teacher writes / / = 6 + 5

7. Teacher writes 2 x 4 = / /

8. Teacher writes / / = 12 - 7

9. Teacher writes 8 + 4 = / /

10. Teacher writes 12 - / / = 3

GIVEN: Eighteen objects are placed in random order before the child.

DIRECTIONS: "Make as many groups of ten as you can. How many ones are left over? Write a numeral showing the number of tens and the number of ones."

PROBLEM:

11.

GIVEN: The exercise 23 - 9 = / /. Teacher groups twenty three objects into 2 sets of ten and 3 ones.

DIRECTIONS: "Using these objects, show how to find the answer to the exercise, 23 - 9 = / /."

PROBLEM:

12. 23 - 9 = / /

GIVEN: Sets of 2, 6, and 3 objects. Teacher joins sets of 2 objects and 6 objects. Then joins a set of 3 objects with the resulting set. Secondly, the teacher joins sets of 6 objects and 3 objects and then joins a set of 2 objects with the resulting set.

DIRECTIONS: "Write a number sentence to show what I have done."

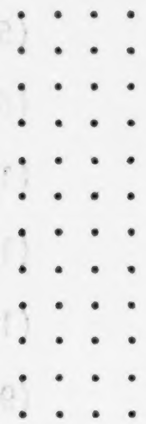
PROBLEM:
13. ANSWER: (2+6)+3=2+(6+3)

GIVEN: A 12 by 4 array of dots.

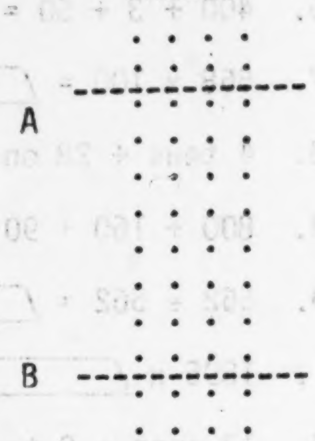
DIRECTIONS: "Use a line to separate the array into two parts as shown by the number sentence."

PROBLEM:

14. $12 \times 4 = (10 \times 4) + (2 \times 4)$



ANSWER



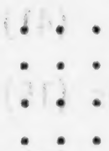
The line may go in position A or B, but not both.

GIVEN: A 4 by 3 array.

DIRECTIONS: "Find as many multiplication and division sentences as you can with the array."

PROBLEM:

15.



POSSIBLE ANSWERS
4x3=12
3x4=12
12÷4=3
12÷3=4

COMPUTATION

WRITE THE ANSWER TO EACH PROBLEM

1. 0 ÷ 1275 = ▱ (1275)

2. 2 tens + 5 ones = ▱ (25)

3. ▱ = 8869 x 1 (8869)

4. 200 + 70 + 5 = ▱ (275)

5. 7689 x 0 = ▱ (0)

6. 400 + 3 + 50 = ▱ (453)

7. 568 x 100 = ▱ (56800)

8. 4 tens + 28 ones = ▱ (68)

9. 800 + 160 + 90 + 4 = ▱ (1054)

10. 562 ÷ 562 = ▱ (1)

11. 1895 x ▱ = 18950 (10)

12. 17 ones + 8 tens = ▱ (97)

13. ▱ = 781 ÷ 1 (781)

14. 75 = 6 tens + ▱ ones (15)

15. 289 + ▱ = 289 (0)

16. 27 tens + 9 ones = ▱ (279)

17. 6500 ÷ 100 = ▱ (65)

18. (12 + 16) + 13 = ▱ (41)

19. 457 = 3 hundreds + ▱ tens + 7 ones (15)

20. ▱ x 1268 = 0 (0)

21. 600 + 4000 + 5 + 70 = ▱ (4675)

22. 765 ÷ ▱ = 1 (765)

23. 97 = ▱ tens + 17 ones (8)

24. 890 ÷ 100 = /______/ (89)

25. 2 hundreds + 9 thousands + 3 ones + 4 tens = /______/
(9243)

WRITE THE NUMERAL NEXT TO THE CORRECT PLACE.
WHAT PLACE VALUE IS HELD BY THE ______

26. 4 in 5467 / / tens / / hundreds / / ones / / thousands

27. 5 in 5321 / / tens / / hundreds / / ones / / thousands

28. 8 in 2783 / / tens / / hundreds / / ones / / thousands

CIRCLE THE CORRECT ANSWER:

29. (10x20)x33= 200 x 33 (20x33)+10 10+(20x33)

30. 9 x 45 = (9x41)+(9x5) (9x40)+(9x6) (9x40)+(9x5)

FIND THE ANSWER TO EACH PROBLEM:

31. 42
-18
(24)

32. 98
+54
(152)

33. 46
x 7
(322)

34. 3/ 42
(14)

35. 467
93
+386
(946)

36. 975
-352
(623)

37. 153
x24
(3672)

38. 923
+187
(1110)

39. 507
-189
(318)

40. 367
x125
(45875)

41. 231
125
+ 6
(362)

42. 923
-547
(376)

WORD PROBLEMS

FIND THE ANSWER TO EACH PROBLEM:
SHOW YOUR WORK IN THE SPACE PROVIDED.

43. Six sisters had 191 sea shells in a box. Carol, one of the sisters, used 67 of them on place cards for a party. How many sea shells were left in the box?

ANSWER

$$\begin{array}{r} 191 \\ -\ 67 \\ \hline 124 \end{array}$$

44. David has 35 stamps. Steve has 62 stamps. Dan has 14 stamps. The 3 boys are brothers and have a hobby of stamp collecting. How many stamps do the three boys have?

ANSWER

$$\begin{array}{r} 35 \\ 62 \\ +14 \\ \hline 111 \end{array}$$

45. Kay put 56 stamps into a book. She collects stamps as a hobby. Kay put 7 stamps on each page of a book. How many pages of the book did Kay use?

ANSWER

$$7 \overline{)\,56}^{\;8}$$

46. Carol collects sea shells as a hobby. One day her mother told her to give some of the shells away. Carol gave five shells to each of eight friends. How many shells did she give away?

ANSWER

$$\begin{array}{r} 8 \\ \times 5 \\ \hline 40 \end{array}$$

STUDENT'S TEST BOOKLET

EXPERIMENTER-DESIGNED ACHIEVEMENT TEST

1.

2.

3.

4.

5.

6. /______/ = 6 + 5

7. /______/ = 12 - 7

8. 2 x 4 = /______/

9. 8 ÷ 4 = /______/

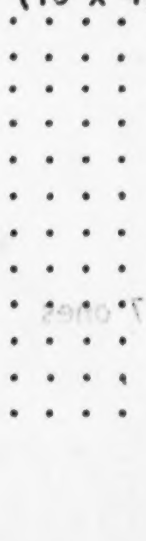
10. 12 - /______/ = 3

11.

12. 23 - 9 = /______/

13.

14. 12 x 4 = (10 x 4) + 2 x 4)



15. ____

COMPUTATION

1. 0 + 1275 = /______/
2. 2 tens + 5 ones = /______/
3. /______/ = 8869 x 1
4. 200 + 70 + 5 = /______/
5. 7689 x 0 = /______/
6. 400 + 3 + 50 = /______/
7. 568 x 100 = /______/
8. 4 tens + 28 ones = /______/
9. 800 + 160 + 90 + 4 = /______/
10. 562 ÷ 562 = /______/
11. 1895 x /______/ = 18950
12. 17 ones + 8 tens = /______/
13. /______/ = 781 ÷ 1
14. 75 = 6 tens + /______/ ones
15. 289 + /______/ = 289
16. 27 tens + 9 ones = /______/
17. 6500 ÷ 100 = /______/
18. (12 + 16) + 13 = /______/
19. 457 = 3 hundreds + /______/ tens + 7 ones
20. /______/ x 1268 = 0

21. 600 + 4000 + 5 + 70 = /______/

22. 765 ÷ /______/ = 1

23. 97 = /______/ tens + 17 ones

24. 890 ÷ 10 = /______/

25. 2 hundreds + 9 thousands + 3 ones + 4 tens = /______/

WRITE THE NUMERAL NEXT TO THE CORRECT PLACE.
WHAT PLACE IS HELD BY THE __________

26. 4 in 5467 ☐ tens ☐ hundreds ☐ ones ☐ thousands

27. 5 in 5321 ☐ tens ☐ hundreds ☐ ones ☐ thousands

28. 8 in 2783 ☐ tens ☐ hundreds ☐ ones ☐ thousands

CIRCLE THE CORRECT ANSWER:

29. (10 x 20) x 33 = 200x33 (20x33)+10 10+(20x33)

30. 9 x 45 = (9x41)+(9x5) (9x40)+(9x6) (9x40)+(9x5)

FIND THE ANSWER TO EACH PROBLEM:

31. $$\begin{array}{r} 42 \\ -18 \\ \hline \end{array}$$

32. $$\begin{array}{r} 98 \\ +54 \\ \hline \end{array}$$

33. $$\begin{array}{r} 46 \\ \times 7 \\ \hline \end{array}$$

34. $3\overline{)42}$

35. $$\begin{array}{r} 467 \\ 93 \\ +386 \\ \hline \end{array}$$

36. $$\begin{array}{r} 975 \\ -352 \\ \hline \end{array}$$

37. $$\begin{array}{r} 153 \\ \times 24 \end{array}$$

38. $$\begin{array}{r} 923 \\ +187 \\ \hline \end{array}$$

39. $$\begin{array}{r} 507 \\ -189 \\ \hline \end{array}$$

40. $$\begin{array}{r} 367 \\ \times 125 \\ \hline \end{array}$$

41. $$\begin{array}{r} 231 \\ 125 \\ +\ 6 \\ \hline \end{array}$$

42. $$\begin{array}{r} 923 \\ -547 \\ \hline \end{array}$$

WORD PROBLEMS

FIND THE ANSWER TO EACH PROBLEM.
SHOW YOUR WORK IN THE SPACE PRVOIDED.

43. Six sisters had 191 sea shells in a box. Carol, one of the sisters, used 67 of them on place cards for a party. How many sea shells were left in the box?

44. David has 35 stamps. Steve has 62 stamps. Dan has 14 stamps. The three boys are brothers and have a hobby of stamp collecting. How many stamps do the three boys have?

45. Kay put 56 stamps into a book. She collects stamps as a hobby. Kay put 7 stamps on each page of a book. How many pages of the book did Kay use?

46. Carol collects sea shells as a hobby. One day her mother told her to give some of the shells away. Carol gave five shells to each of eight friends. How many shells did she give away.

APPENDIX D

Attitude Scales

1. Arithmetic
2. School
3. Reading
4. Spelling
5. Science

DATE ____________________ GRADE ____________________

SCHOOL ____________________ NAME ____________________

DIRECTIONS: (Read silently while your teacher reads to you).

The reason you are filling out this worksheet is to find out how certain words make you feel. When you fill out this worksheet, decide how the word at the top of the page makes you feel, and then mark the scales below the word. Mark the word closest to the way you feel about the word at the top of the page.

Here is how you use the scales:

If the word at either end of the scale very strongly describes your feeling about the word at the top of the page, you should place your check-mark as shown below:

FAIR __X__:_____:_____:_____:_____:_____:_____ UNFAIR

OR

FAIR _____:_____:_____:_____:_____:_____:__X__ UNFAIR

If the word at either end of the scale gives a fairly good description of how you feel about the word at the top of the page (but if you don't feel quite so strongly about the word), place a check-mark as shown below:

GOOD _____:__X__:_____:_____:_____:_____:_____ BAD

GOOD _____:_____:_____:_____:_____:__X__:_____ BAD

If the word at either end of the scale only slightly describes your feeling about the word (your feelings aren't very strong about the word), place a check-mark as shown below:

EASY _____:_____:__X__:_____:_____:_____:_____ HARD

EASY _____:_____:_____:_____:__x__:_____:_____ HARD

If the word at either end of the scale doesn't seem to describe your feelings about the word at the top of the page, place as check-mark as shown below:

WEAK _____:_____:_____:__X__:_____:_____:_____ STRONG

IMPORTANT:

1. Place your check-mark in the middle of the space, not the edge. The correct way is shown below:

_____:_____:__X__:_____:_____:____X:_____

THIS NOT THIS WAY

2. Be sure you check every scale. Do not skip any.

3. Never put more than one check-mark on a single scale.

Keep moving fairly rapidly through this text. Don't stop for too long on any one scale, since we are interested in how you feel when you first look at the words.

SAMPLE EXERCISE

Now you are ready to do a sample exercise. Do not turn the page until your teacher tells you to do so. Work only those scales on the page marked "SAMPLE PAGE". Do not turn the sample page when you have finished with it. Wait for your teacher to tell you.

Remember, look at the word at the top of the page. Then mark each of the scales below. Do not skip any items.

Are there any questions? Ready - Begin.

Note to teacher - After the students have completed the sample page, answer any questions which they may have. Then have them begin the remainder of test by saying, "Ready - Begin".

"SAMPLE PAGE"

VACATION

FAIR ____:____:____:____:____:____:____ UNFAIR

BAD ____:____:____:____:____:____:____ GOOD

SOFT ____:____:____:____:____:____:____ HARD

NOT IMPORTANT ____:____:____:____:____:____:____ IMPORTANT

SMART ____:____:____:____:____:____:____ DUMB

EASY ____:____:____:____:____:____:____ HARD

USEFUL ____:____:____:____:____:____:____ USELESS

WEAK ____:____:____:____:____:____:____ STRONG

BORING ____:____:____:____:____:____:____ FUN

EXCELLENT ____:____:____:____:____:____:____ POOR

FOOLISH ____:____:____:____:____:____:____ WISE

STOP

DO NOT TURN PAGE

Teacher - Please answer any questions before students are told to begin the remaining portion of the worksheet. Examine the sample page of the students to see that everyone is doing the exercise properly, and that every scale has been marked. Then start them on the remaining portion of the worksheet.

ARITHMETIC

FAIR ____:____:____:____:____:____:____ UNFAIR

BAD ____:____:____:____:____:____:____ GOOD

SOFT ____:____:____:____:____:____:____ HARD

NOT IMPORTANT ____:____:____:____:____:____:____ IMPORTANT

SMART ____:____:____:____:____:____:____ DUMB

EASY ____:____:____:____:____:____:____ HARD

USEFUL ____:____:____:____:____:____:____ USELESS

WEAK ____:____:____:____:____:____:____ STRONG

BORING ____:____:____:____:____:____:____ FUN

EXCELLENT ____:____:____:____:____:____:____ POOR

FOOLISH ____:____:____:____:____:____:____ WISE

Go on to next page.

SCHOOL

FAIR _____:_____:_____:_____:_____:_____:_____ UNFAIR

BAD _____:_____:_____:_____:_____:_____:_____ GOOD

SOFT _____:_____:_____:_____:_____:_____:_____ HARD

NOT IMPORTANT _____:_____:_____:_____:_____:_____:_____ IMPORTANT

SMART _____:_____:_____:_____:_____:_____:_____ DUMB

EASY _____:_____:_____:_____:_____:_____:_____ HARD

USEFUL _____:_____:_____:_____:_____:_____:_____ USELESS

WEAK _____:_____:_____:_____:_____:_____:_____ STRONG

BORING _____:_____:_____:_____:_____:_____:_____ FUN

EXCELLENT _____:_____:_____:_____:_____:_____:_____ POOR

FOOLISH _____:_____:_____:_____:_____:_____:_____ WISE

Go on to next page.

READING

FAIR ____:____:____:____:____:____:____ UNFAIR

BAD ____:____:____:____:____:____:____ GOOD

SOFT ____:____:____:____:____:____:____ HARD

NOT IMPORTANT ____:____:____:____:____:____:____ IMPORTANT

SMART ____:____:____:____:____:____:____ DUMB

EASY ____:____:____:____:____:____:____ HARD

USEFUL ____:____:____:____:____:____:____ USELESS

WEAK ____:____:____:____:____:____:____ STRONG

BORING ____:____:____:____:____:____:____ FUN

EXCELLENT ____:____:____:____:____:____:____ POOR

FOOLISH ____:____:____:____:____:____:____ WISE

Go on to next page.

SPELLING

FAIR _____:_____:_____:_____:_____:_____:_____ UNFAIR

BAD _____:_____:_____:_____:_____:_____:_____ GOOD

SOFT _____:_____:_____:_____:_____:_____:_____ HARD

NOT IMPORTANT _____:_____:_____:_____:_____:_____:_____ IMPORTANT

SMART _____:_____:_____:_____:_____:_____:_____ DUMB

EASY _____:_____:_____:_____:_____:_____:_____ HARD

USEFUL _____:_____:_____:_____:_____:_____:_____ USELESS

WEAK _____:_____:_____:_____:_____:_____:_____ STRONG

BORING _____:_____:_____:_____:_____:_____:_____ FUN

EXCELLENT _____:_____:_____:_____:_____:_____:_____ POOR

FOOLISH _____:_____:_____:_____:_____:_____:_____ WISE

Go on to next page.

SCIENCE

FAIR _____:_____:_____:_____:_____:_____:_____ UNFAIR

BAD _____:_____:_____:_____:_____:_____:_____ GOOD

SOFT _____:_____:_____:_____:_____:_____:_____ HARD

NOT IMPORTANT _____:_____:_____:_____:_____:_____:_____ IMPORTANT

SMART _____:_____:_____:_____:_____:_____:_____ DUMB

EASY _____:_____:_____:_____:_____:_____:_____ HARD

USEFUL _____:_____:_____:_____:_____:_____:_____ USELESS

WEAK _____:_____:_____:_____:_____:_____:_____ STRONG

BORING _____:_____:_____:_____:_____:_____:_____ FUN

EXCELLENT _____:_____:_____:_____:_____:_____:_____ POOR

FOOLISH _____:_____:_____:_____:_____:_____:_____ WISE

Stop

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ERIC REPORT RESUME

Resume Date July 1971	Is document copyrighted? ERIC Reproduction release?	No Yes

Title: Differential Effects of Activity-Oriented Vs. Textbook-Oriented Mathematics Instruction for Elementary School Children with Learning and Behavior Problems

Personal Authors: Dunlap, William, Maertens, Norbert, Hotchkiss James M.

Institution (Source): University of Oregon, College of Education, Eugene, Ore.

Publication Date: July 1971 Contract Number: USOE Grant No. OEG-0-9-470117-3664(721)

Pagination: 197

Abstract:

This study compared the effects of two experimental arithmetic treatments upon the achievement and attitude development of fourth grade children. These treatments were called Laboratory and Textbook.

Prior to the study, the experimenter employed task analysis procedures to develop hierarchies of skills for the four operations on whole numbers. Then every skill in the hierarchies was defined by behavioral objectives. After this a diagnostic test was constructed to measure a child's knowledge of each skill in the hierarchies. Furthermore, exercises on both the experimenter-designed diagnostic and achievement tests required children: (1) to manipulate objects illustrating a number sentence, (2) to write a number sentence describing the action performed on objects by an examiner, and (3) to solve problems with paper and pencil. Although the diagnostic test was administered to all children, only the teachers of the Laboratory Treatment used the children's performance as a guide for constructing instructional activities.

Children assigned to the Textbook Treatment performed paper-pencil instructional activities from an arithmetic textbook whose grade level corresponded to their grade-equivalent scores on the Stanford Diagnostic Arithmetic Test. In contrast to this, children assigned to the Laboratory Treatment performed teacher-designed instructional activities which involved the extensive use of games, puzzles, patterns, and the manipulation of physical objects by children.

The instructional phase lasted for 29 days and was administered by teachers participating in the Experienced Teacher Fellowship Program at the University of Oregon. In preparation for the study, all Fellows received the same amount of training. Two Fellows were then randomly assigned to each of twelve elementary schools in the Greater Eugene Metropolitan Area. Within each school, the Fellows identified children with learning difficulties in arithmetic as evidenced by their performance on: (1) standardized arithmetic achievement tests, (2) teacher-constructed tests, and (3) daily arithmetic

assignments. Later, both children and Fellows were randomly assigned to the two treatments. The study involved 147 children with 73 assigned to the Laboratory Treatment and 74 assigned to the Textbook Treatment.

At the conclusion of the study, all children were administered, in this order; attitude scales, Comprehensive Test of Basic Skills: Arithmetic, and the experimenter-designed achievement test. Three measures of children's arithmetic achievement were obtained: (1) computational skills, (2) knowledge of concepts, and (3) application of processes. Further, using Osgood's Semantic Differential as a model, measures were taken of children's attitudes toward arithmetic, school, reading, spelling, and science. Data was analyzed by two-way analysis of variance techniques for a fixed effects model with unequal cell frequencies. An unweighted means procedure was used.

As measured by the commercial achievement test, no significant differences between treatments were found for each of the three areas of arithmetic achievement that were investigated. However, on the experimenter-designed achievement tests, significant differences between treatments were found for concepts and computational skills at the .05 and .01 levels, respectively. The Laboratory Treatment was favored on the concepts section while the Textbook Treatment was favored on the computational skills section. Furthermore, no significant differences between treatments were found for attitudes toward arithmetic school, spelling, and science. Significant differences favoring the Laboratory Treatment were found on the attitude toward reading scale at the .05 level.